Here modern viewpoint: \( F(T^2) \) is split-generated by
2 objects \( \alpha \) and \( \beta \).

Forally, every object \( C \) direct summand in iterated
mapping cone of \( \alpha, \beta \).

Yoneda embedding (contravariant):
\[ C \rightarrow \text{mod-} \mathcal{C} \]
\[ L \rightarrow \{ \text{hom}(T, L) \} \quad \text{for } T \in \mathcal{C} \text{ chain cxs} \]
\[ \text{and str. maps } (A_\alpha) \]

In category of modules, have mapping cones:
given \( f: A \rightarrow B \) closed \( (\mu(f) = 0) \), \( f \in \text{hom}(A, B) \text{ chain } \alpha \)
\[ \text{Cone } (A \rightarrow B) = A^{i+1} \oplus B^i = A[T] \oplus B \]
\[ \overset{\partial_A}{\searrow} \overset{f}{\rightarrow} \overset{\partial_B}{\nearrow} \]

(in Alg. Top., this is what chains on mapping cone look like)

Given \( A, B \in \mathcal{C} \), \( f: A \rightarrow B \) closed, say \( C \in \mathcal{C} \) is
a cone of \( f \) if the module cones to \( C \) is qiso to Cone \( (f) \).

Given a mapping cone, have an exact triangle
\[ A \rightarrow B, \text{ hence an encoupled } L \in \mathcal{C} \text{ AT: } \]
\[ \ldots \rightarrow \text{hom}(T, A) \rightarrow \text{hom}(T, B) \rightarrow \text{hom}(T, C) \rightarrow \ldots \]
\( T^2: \quad \text{cone}(\omega \rightarrow p, p) \rightarrow Y \)  
"Mapping cones are related to surgery."

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\[ \text{Triangulated categories} \]
An exact triangle is  
\[ A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{e} \]

It induces LES for every \( T \)
\[ \ldots \rightarrow H^i \text{hom}(T, A) \xrightarrow{f^*} H^i \text{hom}(T, B) \xrightarrow{g^*} H^i \text{hom}(T, C) \xrightarrow{e^*} \ldots \]
These are natural w.r.t. \( T \).

Can always enlarge Fukaya catecogy so it has mapping cones.
One way is to have twisted complexes (see Seidel’s book)

\[ \text{Tw}(F): \quad \text{Obj.}: \quad \text{finite collection } E = \bigoplus_{i=1}^k E_i \{ \delta_i \} , E_i \in \text{Obj}(F) \]
\[ \text{differential } \delta \in \text{End}^1(E), \text{ i.e } \delta_{ij} \in \text{hom}^{i-j+1} (E_i, E_j) \]

1. \( \mu^i(\delta) + \mu^{i+1}(\delta, \delta) + \ldots = 0 \)
\[ E_1 \rightarrow E_2 \rightarrow E_3 \]

2. \( \delta \) strictly triangular : \( \delta_{ij} = 0 \) unless \( i \leq j \).
\[ (\Rightarrow \text{finiteness in 1}) \]

\[ \text{Ex.}: \quad E_1 \xrightarrow{f} E_2 \xrightarrow{g} E_3 \quad \text{twisted if if}\]
\[ \mu^i(f) = 0, \mu^1(g) = 0, \mu^2(g, f) + \mu^1(f) = 0 \quad \text{for some } E_1 \rightarrow E_3. \]
Hoytis: hom of twisted complexes $\cong$ homs between summands

$$E_1 \xrightarrow{\delta_E} E_2 \xrightarrow{\delta_E} \cdots \xrightarrow{\mu^1_{Tw}(f)} \sum_{k>0} \mu^{k+1} (\delta_{E_f}, \delta_{E_f}, f, \delta_{E_f}, \delta_{E_f})$$

Similarly, given $f_1, \ldots, f_k$ maps between $k+1$ twisted csks,

$$\mu^k_{Tw}(f_1, \ldots, f_k) = \sum \mu (\delta \cdot \delta \cdot f_k \cdot \delta \cdot \delta \cdot f_1 \cdot \delta \cdot \delta)$$

These operations satisfy $A\infty$ relations.

Prop: $Tw(F)$ has mapping cones:

given $(E, \delta_E) \xrightarrow{f} (F, \delta_F)$ s.t. $\mu^1_{Tw}(f) = 0$, then

core $(f) := (E[U] \oplus F, (\delta_E \cdot f, \delta_F))$.

Mapping cones have at least 2 geometric origins in $F$:
1) Dehn twists about Lagr. spheres [Seidel].
2) Lagr. surgery (connected sum):

- $(A \#_p B)$ is a cone of $A \xrightarrow{f} B$
- (FOOO, Dehn twist case in dim 1)
- $(B \#_p A) \neq (A \#_p B)$ (for $f \neq f'$)

- $CF(T, C) = CF(T, A) \oplus CF(T, B)$ in dim $> 1$
- $m^1 \circ (m^2 (\text{coeff} p, \cdot)) \circ U \circ m^1$

For $T^m IR^n$

$T$ coeff accounts for $T^{-E}$ (area difference $\text{ann} \cdot \text{ann}$)
& for difference in local systems

In $Tw(F)$, $C = \{ A \xrightarrow{f} B \}$. 
Ex: $T^2$: $A, B \in \mathcal{F}(T^2)$

In smallest full subal of $T \mathcal{F}(T^2)$ containing $A$ & $B$ & mapping cone, can build curves representing any slope but only balanced w.r.t. $180^\circ$ rotation about curve $C$.

Cone $\left( C \xrightarrow{T^2} B \right) \cong A_1 \oplus A_2$ both isotopic (non- Ham) to $A$.

Every obj of $\mathcal{F}(T^2)$ is $\cong$ to a direct summand in a twisted complex built from $A, B$:

"$A, B$ split-generate $\mathcal{F}(T^2)$".

Note: $\text{Coh}$ is split-closed, but $\mathcal{F}$ is not in general.

E.g: $\text{Coh} \cong \mathbb{P}^2$ splits as two summands that are not geometric (at least in char $0$).

So, need to take split-closure of $\mathcal{F}$.

"To compute $\mathcal{F}(T^2)$, enough to compute for $A, B$, w/ all $A_{\infty}$-structure".

Have functor

$$\mathcal{F}(T^2) \rightarrow \text{mod-}A \ A_{\infty} \text{-modules}$$

$$T \mapsto \text{CF} (T^2, A) \otimes \text{CF}(T^2, B)$$

$A = \text{End}(A \otimes B)$. 
\[ m^3(p, q, r) = 0 \]

\[ \mu^1 = 0 \]
\[ \mu^2(p, q) = f_B \]
\[ \mu^2(q, p) = f_A \]

Levili-Penrose: non-trivial \( \mu^6, \mu^8 \).

Goal: find small collection of generating objects, for which one doesn't have to compute high \( \mu^k \).

Levili-Penrose: A\(_\infty\)-stair on \( A \) are classified by two scalars \( (\mu^6, \mu^8) \).

Mirror elliptic curve: \( y^2 = x^3 + \theta x + \eta \)

Widthens form up to rescaling action

Area \( (T^2) \) \( \rightarrow \) modular parameter of mirror

\( f_T \), \( \omega = \kappa \), B-field...

When puncture \( T^2 \), might expect to not see holomorphic, but still have \( \mu^6, \mu^8 \neq 0 \)!

When place \( \mu \) form periods, there are hexagons & octagons

The calculation is done via \( H^1 \).
Ex: IR x S^1:

\[
\begin{align*}
\Theta & \mapsto \text{Coh}(\mathbb{K}^*) = f.g \text{ mod } \mathbb{K}[X^{\pm 1}] \\
\text{not explicitly shown, if allow non-closed isotopies on left, } x = IR x S^1: (Abouzaid-Seidel)
\end{align*}
\]

Wrapped Fukaya category

Flow thru with hamiltonian perturbation growing quadratically at \(\infty: H = \frac{1}{2}r^2\)

\[\omega = dr \wedge d\Theta, \quad X_h = r d\Theta\]

\[\psi^0(\omega)\]

\[r=0 \quad r=1 \quad r=2\]

\[\text{Coh}(L_0, L_0) = \bigoplus_{h \in \mathbb{Z}} K [x_h] \text{, all in deg } 0\]

\[\Rightarrow \text{ diff } = 0\]

Product:

\[\text{Coh}(L_0, L_0; H) \otimes \text{Coh}(L_0, L_0; H) \to \text{Coh}(L_0, L_0; 2H)\]

For well-definedness, need \(H\) to grow from input to output.

\[\tilde{\nu}(s,t) = \psi^{2-t}_H(\nu(s,t))\]

Time -1 chords \(L_0 \to L_0\) intertwined time -2 chords

\[\cong \psi^1(L_0) \cap L_0 \quad \text{by } r \to 2r\]

Legendre flow
Unwinding: \( q'(x_1) \)

\[ CW^*(L_0, L_0) \cong K[x^{\pm 1}] \]

\[ x_i \leftrightarrow x^i \]

\[ \mu^2(x_i, x_0) = T^{-i} x_i \]

- can absorb \( T^{-i} \) by suitably rescaling generators (by its action)
- \( \mu^2(x_i, x_j) = x_{i+j} \)