# Erratum to "On the ranks of Selmer groups for elliptic curves over $\mathbf{Q}$"

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• In theorem 3.2 of [Ur13], we omit to say that for all $x \in \mathfrak{U}(\overline{\mathbf{Q}}_p)$, the Galois representation $\rho_x$ is poralized in the sense of loc. cit.

• In the proof of Lemma 3.4, we appealed to Lemma 4.2.3 of [SU06b]. In fact, we meant to appeal to a lemma in a earlier and non-published version of [SU06b]. The lemma is in fact the following:

**Lemma** *Let $L$ be a finite extension of $\mathbf{Q}_p$. Let $V$ and $E$ be two $L$-representations of $G_K$ fitting in an exact sequence*

$$0 \to L(1) \to E \xrightarrow{g} V \to 0$$

*Assume $V$ is de Rham and there exists $D' \subset D_{dR}(E)$ such that $g(D') \oplus D^0_{dR}(V) = D_{dR}(V)$.*

*Then $E$ is a de Rham representation. In particular, the class $[E]$ of this extension in $H^1(G_K, V^\vee(1))$ belongs to $H^1_g(G_K, V^\vee(1))$.*

**Proof** We consider the following commutative diagram with exact columns and rows.

$$\begin{array}{ccccccc}
 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \\
 & & D^0_{dR}(E) & \xrightarrow{\sim} & D^0_{dR}(V) & & \\
 & & \downarrow & & \downarrow & & \\
L(1) \otimes_{\mathbf{Q}_p} K.t^{-1} & \longrightarrow & D_{dR}(E) & \longrightarrow & D_{dR}(V) & \xrightarrow{\delta_1} & H^1(K, L(1) \otimes B_{dR}) \\
\downarrow & & \downarrow f_E & & \downarrow f_V & & \downarrow i \\
L(1) \otimes_{\mathbf{Q}_p} K.t^{-1} & \longrightarrow & (E \otimes_{\mathbf{Q}_p} \frac{B_{dR}}{B^+_{dR}})^{G_K} & \longrightarrow & (V \otimes_{\mathbf{Q}_p} \frac{B_{dR}}{B^+_{dR}})^{G_K} & \xrightarrow{\delta_0} & H^1(K, L(1) \otimes \frac{B_{dR}}{B^+_{dR}}) \\
 & & & & \downarrow & & \\
 & & & & 0 & &
\end{array}$$

By a theorem of Sen-Tate, we have $H^0(K, tB^+_{dR}) = H^1(K, tB^+_{dR}) = 0$ which explains the isomorphism of the first row of the diagram and the fact that $i$ is injective. Note also that the third column is exact because $V$ is de Rham.

Since $g(D') \oplus D^0_{dR}(V) = D_{dR}(V)$, $g(f_E(D')) = f_V(g(D')) = D_{dR}(V)/D^0_{dR}(V) = (V \otimes \frac{B_{dR}}{B^+_{dR}})^{G_K}$ and $\delta_0 = 0$. This implies $\delta_1 = 0$ and therefore $E$ is de Rham.

□

# References


[SU06b] C. M. Skinner and E. Urban, *Vanishing of L-functions and ranks of Selmer groups*, 28 pages, Proceedings of the International Congress of Mathematician held in Madrid 2006, vol. II, pp 473–500.

[Ur13] E. Urban, *On the ranks of Selmer groups of elliptic curves over* **Q**, in *Automorphic Representations and L-Functions*, Proceedings of the International Colloquium held at the Tata Institute of Fundamental Research, pp 651– 680 (2013)



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