IN CLASS PRACTICE PROBLEMS

THE MAGIC OF NUMBERS

(1) Recall that $5! = 5 \times 4 \times 3 \times 2 \times 1$ and $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$, the product of it and all the (positive) whole numbers less than it. For example, $11! = 39916800$ and has 2 zeros at the end of the number. How many zeros are there at the end of 101!?

(2) [1, 133] How many positive integers have decimal representations consisting of distinct digits?

(3) A combination door lock has 6 buttons, numbered from 1 to 6. Once a button is pushed, it stays down and cannot be pushed again. It is not necessary that all buttons get pushed, i.e. you can have a 1,2,3,4,5 or 6 digit code.
   (a) How many valid combinations are there?
   (b) Suppose we allow buttons to be pushed in pairs of two, the order within a pair is irrelevant since they are being pushed simultaneously, and we require that combinations be 6 digits long. E.g. 63 - 15 - 24 is the same as 63 - 15 - 42 but different from 15 - 63 - 24 and 12 - 23 - 56 is impossible because 2 appears twice. How many combinations are there now?
   (c) In the previous problem, how many combinations are there if we allow combinations of length 2, 4 or 6 digits?
   (d) [1, 135] Suppose we allow buttons to be pushed in groups, with each group being a simultaneous push of 1, 2, 3, 4, 5 or 6 buttons and the order within a group is irrelevant since they are being pushed simultaneously. E.g. 3 - 15 - 24 is the same as 3 - 15 - 42 but different from 15 - 3 - 24 and 12 - 23 is impossible because 2 appears twice. How many valid combinations are there now?

(4) **Birthday Problem** How many people are necessary to ensure that there is a 50% probability that there are two people that share a birthday. Assume that birthdays are equally distributed over a 365 day year.

(5) [3] You know that your boss has two children and at least one of them is a boy.
   (a) What is the probability that both of them are boys?
   (b) Suppose that you knew that the older one is a boy. Does this change the probability? What if you knew that the younger one is a boy?
   (c) [4] Suppose you run into your boss and he/she is with his/her son. What is the probability that your boss has two boys?

(6) [2, 5.3.3] What is the probability that a 5 card hand will have, either all red or all black cards? Is this the same as the odds of 5 coins coming up all heads or all tails? Why or why not?

(7) [4] Suppose you discover an unfair coin, i.e. if you flip it there is not a 50% probability that it comes back as Heads - the probability is either higher or lower. You and your friend need to solve a dispute and need a fair way to choose a winner. Could you devise a process (involving just this coin) that gives you a way to choose a winner? Note: you don’t need to know the actual probability of getting a Head or Tails.

Hint:¹

¹Think about using multiple flips.
References


