**PROBLEM SET 4**

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**Exercise 0.1.** Let $V \subset \mathbb{A}^n_k$ be an affine algebraic set. Let $I(V/k) := I(V) \cap k[X]$. Show that $V = V(J)$ where for some subset $J \subset k[X]$ if and only if $I(V/k)\bar{k}[X] = I(V)$.

**Exercise 0.2.** Let $f \in \bar{k}[X_1, \cdots, X_n]$. Show that $V = V(f) \subset \mathbb{A}^n$ is a variety if and only if $f$ is irreducible.

**Exercise 0.3.** Let $V \subset \mathbb{P}^n_k$ be a projective variety. Let $f : V \to \bar{k}$ be a function such that for all $i$, $f|_{V \cap U_i} : V \cap U_i \to \bar{k}$ is given by an element in $\bar{k}[V \cap U_i]$. Show that $f$ has to be a constant function.

**Exercise 0.4.** Let $V = V(Y^2Z - X^3 + X^2Z)$. This is the projective closure of the affine curve $Y^2 = X^3 - X^2$ that is singular at $X = Y = 0$. Let $F = [X/Y : 1] : V \to \mathbb{P}^1_k$. Then $F$ is not regular at $[0 : 0 : 1]$.

**Exercise 0.5.** Let $V = \mathbb{P}^2_k$. Consider the rational map $F = [X/Y : 1] : V \to \mathbb{P}^1_k$. It is not regular at $p = [0 : 0 : 1]$. Hint: Suppose $F$ is regular at $p$. Then for any curve $C$ through $p$, $F$ restricts to a rational map $C \to \mathbb{P}^1$ that is also regular at $p$, with its value at $p$ the same as $F(p)$. Consider the curves $X = Y^2, Y = X^2$. 