Calculus I, Fall 2014
Practice Exam I

September 25, 2014

Please show all work. Each problem is worth 12 points.

1. Find the limit \( \lim_{x \to 3} \frac{x^2+x-12}{x^2+2x-15} \).

2. For which value of \( a \) does the limit \( \lim_{x \to 3} \frac{\sqrt{x^2+16}-a}{x-3} \) exist? What is the value of the limit in this instance?

3. Consider the function \( f(x) \) defined as follows:

\[
f(x) = \begin{cases} 
\frac{x+\pi}{x+8} & \text{if } x \leq -\pi \\
\frac{\sin x}{x} & \text{if } -\pi < x < 0 \\
\frac{x^2+1}{x-1} & \text{if } x > 0 
\end{cases}
\]

At which points is \( f(x) \) discontinuous? Justify your answer.

4. Find the limit \( \lim_{x \to \infty} \frac{3x}{x^2+2x} \).

5. Give an example of a function which is discontinuous at infinitely many points.

6. Find the limit \( \lim_{x \to 1} \frac{\sin(x^2-1)}{x-1} \).

7. a. Calculate the derivative of \( 2x^2 + 3 \), using the definition of the derivative.
    b. For which value of \( b \) is the line \( y = 12x + b \) tangent to the curve \( y = 2x^2 + 3 \)?

8. Suppose the number of badgers in the Central Park Zoo is given by the formula \( b(t) = 40\sqrt{t-1988} + 3 \), where \( t \) denotes time measured in years. Approximately how many badgers did the zoo acquire in the year 2013? (Hint: The slope of the line tangent to the graph of \( y = b(t) \) at any given point \((a, b(a))\) is a good estimate for the rate of badger acquisition at the time \( a \).)

**Bonus (5 points):** Is it true that if \( \lim_{x \to \infty} f(x) = 0 \), then \( \lim_{x \to \infty} f'(x) \) as well? Why or why not?