1. Prove the “extended Euclid lemma”: Given a prime $p$ and nonzero integers $a_1, \ldots, a_k \in \mathbb{Z}$, then $p|(a_1 \cdot a_2 \cdot \ldots \cdot a_k)$ implies that $p|a_i$ for some $1 \leq i \leq k$.

2. Prove that for any $n \in \mathbb{N}^+$, any prime $p$ such that $p|(n! + 1)$ satisfies $p > n$. (Hint: Use problem 1 somehow.) Deduce from this another proof of the infinitude of primes.

3. Given nonzero integers $a, b \in \mathbb{Z}$, prove that a given $c \in \mathbb{Z}$ can be written in the form $c = ma + nb$ for some $m, n \in \mathbb{Z}$ if and only if $(a, b)|c$.

4. Given nonzero integers $a, b \in \mathbb{Z}$, the least common multiple of $a$ and $b$ is the smallest positive integer $n$ such that $a|n$ and $b|n$. Write $\text{lcm}(a, b)$ for this integer.
   i) Prove that $\text{lcm}(a, b)$ is well-defined. (Hint: Use the least element principle.)
   ii) Prove that $\text{lcm}(a, b) = \frac{ab}{(a, b)}$.

5. In the decimal expansion of $99!$, how many zeros does it end with?