Under standard conditions, the graph of an optimal transport map is a calibrated submanifold in the product manifold with an explicit pseudometric. These graph submanifolds are in many ways similar to special Lagrangian submanifolds of Calabi-Yau manifolds. We will discuss some recent progress which follows this observation, including joint work with Kim, McCann and Streets.