Written Homework 3

Due Thursday, October 17th at 6:00 PM in homework box.

Problems:

1. (5 points) The Binomial Theorem states that

\[(x + h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \cdots + \frac{n(n-1)}{2}x^2h^{n-2} + nxh^{n-1} + h^n,\]

where \(x\) and \(h\) can be any real numbers and \(n\) is a positive integer. Use this theorem to prove that if \(f(x) = x^n\), then \(f'(x) = nx^{n-1}\) using the following definition of the limit:

\[f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}.\]

2. (a) (3 points) Use the definition of the derivative (either the “\(h \to 0\)” or the “\(z \to x\)” version is fine) to compute \(f'(x)\) where \(f(x) = \frac{1}{x^3}\).

(b) (1 point) Use your result from part (a) to find the equation for the tangent line to \(f(x)\) at the points \(x = 1\) and \(x = -1\).

(c) (1 point) Sketch \(f(x)\) as well the two tangent lines from part (b) clearly on one graph. What relationship do you find between the tangent lines?