Title : Lipschitz geometry of complex surfaces: analytic invariants and equisingularity

Abstract : The question of defining a good notion of equisingularity of a reduced hypersurface $\mathfrak{X} \subset \mathbb{C}^n$ along a non singular complex subspace $Y \subset \mathfrak{X}$ in a neighbourhood of a point $0 \in \mathfrak{X}$ has a long history which started in 1965 with works of Zariski. One of the central concepts introduced by Zariski is the algebro-geometric equisingularity, called nowadays Zariski equisingularity, which defines the equisingularity inductively on the codimension of Y in $\mathfrak{X}$ by requiring that the reduced discriminant locus of a suitably general projection $p\colon \mathfrak{X} \to \mathbb{C}^{n-1}$ be itself equisingular along $p(Y)$.

When Y has codimension one in $\mathfrak{X}$, *i.e.*, when dealing wih a family of plane curves transversal to the parameter space Y, it is well known that Zariski equisingularity is equivalent to the main notions of equisingularity such as Whitney conditions for the pair $(\mathfrak{X} \setminus Y, Y)$ and topological triviality. However these properties fail to be equivalent in higher codimension.

I will present a recent joint work with Walter Neumann in which we prove that in codimension 2, for a family of hypersurfaces in $\mathbb{C}^3$ with isolated singularities, Zariski equisingularity is equivalent to the constancy of the family up to bilipschitz semi-algebraic homeomorphism with respect to the outer metric.