ABSTRACT

THE HYPERBOLIC GEOMETRY OF SKEW CONVEX HEXAGONS AND $PSL(2,\mathbb{C})$ DISCRETENESS SEQUENCES

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We present a new approach to the $PSL(2,\mathbb{C})$ discreteness problem. A subgroup, G, of $PSL(2,\mathbb{C})$ (equivalently $Isom(\mathbb{H}^3)$) is not discrete if there exists an infinite sequence of distinct elements of the group that converges to the identity. To date there are only ad hoc techniques for finding such a sequence of primitive elements in any given G. When G is the image of a non-elementary representation of a rank two free group, it may or may not be discrete or free, but the representation determines a set of so called *core points* in $\mathbb{H}^3$ and a new ordering of the rational numbers, the *representation ordering*. We use the hyperbolic geometry of $\mathbb{H}^3$ as applied to palindromes in G and the representation ordering of the rationals to construct a unique sequence of primitive elements corresponding to a given representation. Theorem: (i) The sequence of core points is finite if and only if the group is discrete and free (ii) if the sequence is infinite and converges to a point on the boundary of $\mathbb{H}^3$, the group is either not geometrically finite or not discrete and (iii) if the sequence is infinite and converges to an interior point of $\mathbb{H}^3$, the group is not discrete. The proof involves an extension of Fenchel's theory of right angled hexagons in $\mathbb{H}^3$ to *skew-convex* hexagons. This is joint work with Linda Keen.