KNOTS WITH SMALL RATIONAL GENUS

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Abstract. If K is a rationally null-homologous knot in a 3-manifold M then there is a compact orientable surface S in the exterior of K whose boundary represents $p[K]$ in $H_1(N(K))$ for some $p > 0$. We define $\|K\|$, the *rational genus* of K, to be the infimum of $-\chi^-(S)/2p$ over all S and p. If M is a homology sphere then this is essentially the genus of K. By doing surgery on knots in S^3 one can produce knots in 3-manifolds with arbitrarily small rational genus. We show that such knots can be characterized geometrically. More precisely we show that there is a positive constant C such that if K is a knot in a 3-manifold M with $\|K\| < C$ then (M, K) belongs to one of a small number of classes; for example, M is hyperbolic and K is a core of a Margulis tube, M is Seifert fibered and K is a fiber, K lies in a JSJ torus in M, etc. Conversely we show that there are pairs (M, K) in each of these classes with $\|K\|$ arbitrarily small. This is joint work with Danny Calegari.