

GROUPS OF MAPPING CLASSES THAT CANNOT BE REALIZED BY DIFFEOMORPHISMS

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ABSTRACT. Morita proved that the mapping class group cannot be realized by diffeomorphisms. The mapping class group of a surface Σ with one marked point z fits into the short exact sequence

$$1 \rightarrow \pi_1(\Sigma, z) \rightarrow \text{Map}(\Sigma, z) \rightarrow \text{Map}(\Sigma) \rightarrow 1.$$

The kernel is known as the point-pushing subgroup, since its elements are obtained by “pushing” the marked point along loops in the fundamental group of Σ . By using Milnor’s inequality for the Euler number of a flat vector bundle over a surface, we show that the point-pushing subgroup cannot be realized by diffeomorphisms of Σ fixing z . We apply this result to construct a group isomorphic to $\pi_1(\Sigma') \times \mathbb{Z}/3\mathbb{Z}$ inside $\text{Map}(\Sigma)$ that cannot be realized by diffeomorphisms; as a corollary, this yields a new proof of Morita’s theorem. I will conclude by relating this group to the branched surface bundles constructed by Atiyah and Kodaira.

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