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ACTIONS OF FINITE CYCLIC GROUPS ON QUASICOMPLEX MANIFOLDS

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Abstract. In this paper a classification is given of actions of finite cyclic groups on quasicomplex manifolds in terms of the invariants of cobordism theory. Moreover, the methods of the paper allow one to understand the geometric nature of known results of a series of authors on actions of cyclic groups of prime order.

Bibliography: 11 items.

Introduction

As was first remarked in [1], the invariants of bordism theory are always useful for describing fixed points of actions of a compact Lie group G.

In the present paper, by actions and mappings, if nothing is said to the contrary, one understands infinitely smooth actions and mappings which preserve the complex structure in the stable tangent bundle.

In what follows, U_*^G will denote the unrestricted module of G-bordisms (cf. [1], §21).

Singular points of an action of the group G on the manifold M are points $m \in M$ whose isotropy subgroup is nontrivial. *Fixed points* are those singular points whose isotropy subgroup coincides with G.

The set of fixed points is a disconnected union of smooth submanifolds whose normal bundles are complex G-bundles.

We consider vector G-bundles over trivial G-manifolds which cannot be represented as the sum of two vector G-bundles on one of which the action of the group G is trivial. We introduce in the class of these bundles the natural bordism relation. If U_* is the ring of unitary bordisms, then a U_*-module structure is given by multiplication of fiber spaces over a quasicomplex manifold. The U_*-module obtained in this way will be denoted by R_*^G. The grading is given by the real dimension of the fiber space.

The map which associates with a G-manifold the collection of G-bundles normal to the fixed submanifolds gives a homomorphism of graded U_*-modules $\beta^G\colon U_*^G \to R_*^G$.

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Collections of G-bundles which are collections of bundles normal to fixed submanifolds of actions of the group G, i.e. which belong to Im β^G, will be called *admissible*. For the group $\mathbf{Z}_p$ (p prime) admissible collections of fixed submanifolds with trivial normal bundle were found in [2]–[4]; a proof which does not use the techniques of formal groups was obtained in [5]. With the use of the methods of the paper [6] the answer for arbitrary normal bundles was found in [7].

The basic goal of §1 is the description of the admissible collections of normal G-bundles to the fixed submanifolds of actions of the groups $\mathbf{Z}_{p^k}$; however, the method of proof allows one in addition to obtain a more geometric interpretation of the results of [8].

Reduction to groups of the form $\mathbf{Z}_{p^k}$ allows us in §2 to obtain necessary and sufficient conditions on the fixed submanifolds of actions of finite cyclic groups of arbitrary order.

In §3 a homomorphism $\gamma_p^k\colon R_*^{\mathbf{Z}_{p^k}} \to U_* \otimes Q$ is constructed such that its value on an admissible collection coincides modulo the ideal pU_* with the bordism class of the manifold which realizes this collection.

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§1. Admissible collections of fixed submanifolds of actions of the group $\mathbf{Z}_{p^k}$

1.1. By a *representation* of a group G in what follows will be meant a vector space with an action of G on it given by some linear representation of G. The direct sum gives the structure of a semigroup to the set of isomorphism classes of representations. This semigroup is generated by the set of irreducible representations Δ_j, $j \in J(G)$.

For an arbitrary vector G-bundle over a trivial G-manifold there exists a decomposition

$$\zeta = \bigoplus_j (\zeta_j \otimes \Delta_j),$$

where $\zeta_j = \operatorname{Hom}_G(\zeta, \Delta_j)$ (Proposition 2.2 of [9]). An action of G on the bundle $\zeta_j \otimes \Delta_j$ is given by the action of G on the second factor. This decomposition gives an isomorphism of modules

$$R_*^G = \bigoplus U_*\Big(\prod_j \mathbf{BU}(n_j)\Big),$$

the sum being taken over collections $(n_1, \cdots, n_j, \cdots)$ such that all but a finite number of the n_j are equal to zero.

The map $\mathbf{BU}(n) \times \mathbf{BU}(m) \to \mathbf{BU}(n+m)$ introduces a multiplicative structure in $\bigoplus U_*(\Pi_j \mathbf{BU}(n_j))$. Since $U_*(\mathbf{BU}(n))$ is the polynomial ring in variables $(\mathbf{CP}^n)$ with coefficients in U_* ($(\mathbf{CP}^n) \in U_*(\mathbf{CP}^\infty)$ is the bordism class corresponding to the imbedding of the manifold $\mathbf{CP}^n$ in $\mathbf{CP}^\infty = \mathbf{BU}(1)$), the module R_*^G is isomorphic to the polynomial ring with coefficients in U_* in the variables $(\mathbf{CP}_j^n)$, where $(\mathbf{CP}_j^n)$ denotes the G-bundle $\xi \otimes \Delta_j$ over $\mathbf{CP}^n$, ξ being the canonical bundle. Generators of R_*^G as a U_*-module are the monomials $(\mathbf{CP}_{j_1}^{n_1}) \times \cdots \times (\mathbf{CP}_{j_r}^{n_r})$.

All irreducible representations of the group $\mathbf{Z}_m$ are one-dimensional. A generator of $\mathbf{Z}_m$ acts on C^1 by multiplication by $e^{2\pi ij/m}$. We denote the corresponding representation by Δ_j^1. The set of Δ_j^1, $1 \leq j \leq m-1$, is the set of all irreducible representations of the group $\mathbf{Z}_m$.

1.2. We consider a vector $\mathbf{Z}_{p^k}$-bundle with free action of the group $\mathbf{Z}_{p^k}$ outside the zero section, whose base is the singular submanifold of a fiber space. In the class of such bundles (analogous to what was done to introduce the module R_*^G) we define a module $\mathfrak{M}_*^k$. We associate with a vector $\mathbf{Z}_{p^k}$-bundle ζ, whose bordism class belongs to $\mathfrak{M}_*^k$, the collection of $\mathbf{Z}_{p^k}$-bundles normal to fixed submanifolds in the fiber space. This correspondence determines a homomorphism $\mathfrak{M}_*^k \to R_*^{\mathbf{Z}_{p^k}}$. The kernel of this homomorphism is the submodule $\hat{\mathfrak{M}}_*^k$ of bordism classes of bundles in whose total space there are no fixed points. Then χ is the homomorphism

$$\chi : \mathfrak{M}_*^k/\hat{\mathfrak{M}}_*^k \to R_*^{\mathbf{Z}_{p^k}}.$$

The subgroups of $\mathbf{Z}_{p^k}$ are directed by inclusion, so the set of singular points of the action of $\mathbf{Z}_{p^k}$ coincides with the set of fixed points of the action of the subgroup $\mathbf{Z}_p \subset \mathbf{Z}_{p^k}$. This means it is a disconnected union of submanifolds, while the collection of normal $\mathbf{Z}_{p^k}$-bundles obviously determines a bordism class which belongs to the module $\mathfrak{M}_*^k$. The composition of the corresponding homomorphism $U_*^{\mathbf{Z}_{p^k}} \to \mathfrak{M}_*^k$ and the projection $\mathfrak{M}_*^k \to \mathfrak{M}_*^k / \hat{\mathfrak{M}}_*^k$ we shall denote by δ,

$$\delta : U_*^{\mathbf{Z}_{p^k}} \to \mathfrak{M}_*^k/\hat{\mathfrak{M}}_*^k.$$

From the definition of the homomorphism expounded above it follows that $\beta^{\mathbf{Z}_{p^k}} = \chi \circ \delta$. This means that the bordism class $r \in R_*^{\mathbf{Z}_{p^k}}$ belongs to $\operatorname{Im} \beta^{\mathbf{Z}_{p^k}}$ if and only if it belongs to the image of the monomorphism χ and $\chi^{-1}(r)$ belongs to the image of δ.

1.3. For any bundle ζ, $[\zeta] \in \mathfrak{M}_{2n}^k$, a free action of the group $\mathbf{Z}_{p^k}$ on the sphere bundle determines a bordism class $\alpha(\zeta)$ belonging to $\tilde{U}_{2n-1}(\mathbf{BZ}_{p^k})$. Then $\alpha: \mathfrak{M}_*^k \to \tilde{U}_*(\mathbf{BZ}_{p^k})$ is the corresponding homomorphism of graded U_*-modules of degree -1. It is easy to verify the exactness of the sequence

$$U_*^{\mathbf{Z}_{p^k}} \to \mathfrak{M}_*^k \xrightarrow{\alpha} \tilde{U}_*(\mathbf{BZ}_{p^k}).$$

This means that to describe $\operatorname{Im} \delta$, it suffices to study $\operatorname{Ker} \alpha$.

The restriction of the action of the representation Δ to the unit sphere S^{2n-1} of the representation space will also be denoted by Δ. In what follows, $\mathbf{BZ}_{p^k}$ will be represented as the limit of the inclusions of the factor manifolds S^{2n-1}/Δ_1^n, where Δ_j^n is the n-dimensional representation equal to $n \cdot \Delta_j^1$. The inclusion of S^{n-1}/Δ_1^n in $\mathbf{BZ}_p$ we shall denote by i_n. We remark that by definition $\alpha(\Delta_1^n) = [S^{2n-1}/\Delta_1^n, i_n]$.

Let l be a trivial l-dimensional bundle with action of the group $\mathbf{Z}_{p^k}$ in the fiber given by the representation Δ_1^l. The free action of the group on the sphere bundle of the vector $\mathbf{Z}_{p^k}$-bundle $\zeta \oplus l$, $[\zeta] \in \mathfrak{M}^k_{2n}$, gives the bordism class $\widetilde{\alpha}(\zeta \oplus l)$ of the $(2(n+l)-1)$-dimensional skeleton of $\mathbf{BZ}_{p^k}$.

For the manifold S^{2N-1}/Δ_1^N one has the duality isomorphism

$$D_N : U_i(S^{2N-1}/\Delta_1^N) \to U^{2N-i-1}(S^{2N-1}/\Delta_1^N).$$

From Theorem 35.2 of [1] it is easy to get that there exists a homomorphism $\mathrm{D}\alpha\colon \mathfrak{M}^k_* \to U^0(\mathbf{BZ}_{p^k})$ such that $i^*_{n+l}\mathrm{D}\alpha(\zeta) = \mathrm{D}_{n+l}\widetilde{\alpha}(\zeta \oplus l)$.

Lemma. *The bordism class* $[\zeta] \in \mathfrak{M}^k_{2n}$ *belongs to* Ker α *if and only if* $i^*_{n+1}(u \cdot \mathrm{D}\alpha(\zeta)) = 0$, *where* u *is the Euler class of the canonical bundle over* $\mathbf{BZ}_{p^k}$.

Proof. In what follows we shall not distinguish in the notation between a vector bundle over $\mathbf{BZ}_{p^k}$ and its restriction to a finite-dimensional skeleton of $\mathbf{BZ}_{p^k}$, and likewise for the Euler classes of these bundles.

We recall that the cobordism ring $U^*(\mathbf{BZ}_{p^k})$ is isomorphic to $U^*[[u]]/([u]_{p^k} = 0)$, $[u]_{p^k}$ is the p^kth power in the formal group of "geometric" cobordism, and

$$U^*(S^{2N-1}/\Delta_1^N) = U^*[[u]]/([u]_{p^k} = 0,\ u^N = 0).$$

The inclusion $i_N^1\colon S^{2N-1}/\Delta_1^N \to S^{2N+1}/\Delta_1^{N+1}$ determines the bordism class $[S^{2N-1}/\Delta_1^N, i_N^1] \in U_{2N-1}(S^{2N+1}/\Delta_1^{N+1})$, while $\mathrm{D}_{N+1}[S^{2N-1}/\Delta_1^N, i_N^1] = u$. From this fact and the well-known identity $f_*(f^*(a) \cap b) = a \cap f_*(b)$ follows the commutativity of the diagram

$$\begin{array}{ccc} U_{2j-1}(S^{2N-1}/\Delta_1^N) & \xrightarrow{i^1_{N*}} & U_{2j-1}(S^{2N+1}/\Delta_1^{N+1}) \\ \downarrow D_N & & \downarrow D_{N+1} \\ U^{2(N-j)}(S^{2N-1}/\Delta_1^N) & \rightarrow & U^{2(N-j)+2}(S^{2N+1}/\Delta_1^{N+1}), \end{array}$$

where the lower arrow corresponds to the inclusion under which u goes to u, and multiplication by u. The assertion we are proving follows from the fact that $i_{N+1_*}\colon U_{2j-1}(S^{2N+1}/\Delta_1^{N+1}) \to U_{2j-1}(\mathbf{BZ}_{p^k})$ is an isomorphism for $j \le N$.

Remark. One can suggest an equivalent formulation of the lemma, namely the following: $[\zeta] \in$ Ker α *if and only if* $u \cdot \mathrm{D}\alpha(\zeta)$ *is divisible by* u^{n+1} *in the ring* $U^*(\mathbf{BZ}_{p^k})$.

1.4. We associate with each representation Δ of the group G a vector bundle $\nu(\Delta)$ over $\mathbf{BG}$. If C^n is the space of the representation Δ, then on the product $C^n \times \mathbf{EG}$ the group G acts diagonally. Then $\nu(\Delta)\colon (C^n \times \mathbf{EG})/G \to \mathbf{BG}$. For brevity we shall write $e(\Delta)$ for the Euler class of the bundle $\nu(\Delta)$. The ideal of the ring $U^*(\mathbf{BG})$ generated by classes which are annihilated by multiplication by the Euler class of some representation, we shall denote by $I^*(G)$.

Theorem. *The homomorphism* $\mathbf{D}\alpha$ *maps* $\hat{\mathfrak{M}}^k_{2n}$ *epimorphically onto the ideal* $I^0(\mathbf{Z}_{p^k})$.

Proof. Let M be the base of the vector $\mathbf{Z}_{p^k}$-bundle ζ which is a representative of some bordism class $[\zeta] \in \hat{\mathfrak{M}}^k_{2n}$. By definition of the module $\hat{\mathfrak{M}}^k_*$ the action of $\mathbf{Z}_{p^k}$ on M has no fixed points. Then there exists a continuous $\mathbf{Z}_{p^k}$-equivariant map of M into S^{2n-1} with the action $\Delta^n_{p^{k-1}}$. In fact, the factor group $\mathbf{Z}_{p^k}/\mathbf{Z}_{p^{k-1}} = \mathbf{Z}_p$ acts freely on the factor complex $M/\mathbf{Z}_{p^{k-1}}$ and on the sphere S^{2n-1}. Since the dimension of M is less than $2n-1$, there exists a continuous $\mathbf{Z}_{p^k}$-equivariant map of $M/\mathbf{Z}_{p^{k-1}}$ into S^{2n-1}. The composition of it with the projection $M \to M/\mathbf{Z}_{p^{k-1}}$ gives the required map $f: M \to S^{2n-1}$.

Let $S(\zeta \oplus l)$ be the sphere bundle of the vector $\mathbf{Z}_{p^k}$-bundle $\zeta \oplus l$. Since the action of $\mathbf{Z}_{p^k}$ on $S(\zeta \oplus l)$ is free, there exists a $\mathbf{Z}_{p^k}$-equivariant map $g: S(\zeta \oplus l) \to S^{2(n+l)-1}$ into the sphere with the action Δ^{n+l}_1. The projection of the vector $\mathbf{Z}_{p^k}$-bundle $\zeta \oplus l$, and also the corresponding projection of its sphere bundle, we shall denote by p. If h is the map obtained by smoothing the continuous map

$$(f \circ p * g)/\mathbf{Z}_{p^k}: S(\zeta \oplus l)/\mathbf{Z}_{p^k} \to (S^{2n-1} \times S^{2(n+l)-1})/\Delta^n_{p^{k-1}} \times \Delta^{n+l}_1,$$

then by construction

$$\tilde{p}_*[S(\zeta \oplus l)/\mathbf{Z}_{p^k}, h] = \tilde{\alpha}(\zeta \oplus l).$$

We note that $(S^{2n-1} \times S^{2(n+l)-1})/\Delta^n_{p^{k-1}} \times \Delta^{n+l}_1$ is the sphere bundle of the vector bundle $\nu(\Delta^n_{p^{k-1}})$, and $\tilde{p}: \nu(\Delta^n_{p^{k-1}}) \to \mathbf{BZ}_{p^k}$. From the last equation and the exactness of the bordism sequence of the pair consisting of the fiber space $\mathbf{E}\rho_n$ ($\rho_n = \nu(\Delta^n_{p^{k-1}})$) and the sphere bundle $\mathbf{S}\rho_n$ it follows that

$$j_* s_{0*}\tilde{\alpha}(\zeta \oplus l) = 0, \text{ where } j_*: U_*(\mathbf{E}\rho_n) \to U_*(\mathbf{E}\rho_n, \mathbf{S}\rho_n),$$

and $s_0: S^{(2n+l)-1}/\Delta^{n+l}_1 \to \mathbf{E}\rho_n$ is the zero section. The homomorphism $D = \tilde{p}^* D_{n+l} \tilde{p}_*$ is an isomorphism, $D: U_*(\mathbf{E}\rho_n) \to U^*(\mathbf{E}\rho_n)$. The duality isomorphism of the relative bordism of the pair $\mathbf{E}\rho_n$, $\mathbf{S}\rho_n$ and cobordism of $\mathbf{E}\rho_n$ we denote by D',

$$D': U_*(\mathbf{E}\rho_n, \mathbf{S}\rho_n) \to U^*(\mathbf{E}\rho_n).$$

It is easy to verify that the homomorphism $U^*(\mathbf{E}\rho_n) \to U^*(\mathbf{E}\rho_n)$ given by the multiplication by $e(\Delta^n_{p^{k-1}}) = e(\rho_n)$ makes the following diagram commutative:

$$\begin{array}{ccc} U_*(\mathbf{E}\rho_n) & \xrightarrow{j_*} & U_*(\mathbf{E}\rho_n, \mathbf{S}\rho_u) \\ \downarrow D & & \downarrow D' \\ U^*(\mathbf{E}\rho_n) & \longrightarrow & U^*(\mathbf{E}\rho_n). \end{array}$$

Hence it follows that

$$\mathbf{D}_{n+l}\widetilde{\alpha}(\zeta \oplus l) \cdot e(\Delta^n_{p^{k-1}}) = 0 \quad \text{or} \quad \mathbf{D}\alpha(\zeta) \cdot e(\Delta^n_{p^{k-1}}) = 0.$$

This means that $\mathbf{D}\alpha(\zeta) \in I^0(\mathbf{Z}_{p^k})$.

We shall show that $I^*(\mathbf{Z}_{p^k})$ coincides with the ideal generated by the series $\theta_p([u]_{p^{k-1}}) = [u]_{p^k}/[u]_{p^{k-1}}$, where $\theta_p(u)$ is the series equal to $[u]_p/u$. From the structure of the set of irreducible representations of the group $\mathbf{Z}_{p^k}$ it follows that $e(\Delta^1_j)$ for any j divides $e(\Delta^n_{p^{k-1}})$. This means that the Euler class of any n-dimensional representation divides $e(\Delta^n_{p^{k-1}}) = [u]^n_{p^{k-1}}$. Let $P(u) \in U^*[[u]]$ be a representative of some cobordism class of $I^*(\mathbf{Z}_{p^k})$; then

$$P(u) \cdot [u]^n_{p^{k-1}} = [u]_{p^k} Q(u).$$

We divide both sides by $[u]_{p^{k-1}}$:

$$P(u)[u]^{n-1}_{p^{k-1}} = \theta_p([u]_{p^{k-1}})\, Q(u).$$

If $n > 1$, then $pQ(u) = 0 \,(\mathrm{mod}\,[u]_{p^{k-1}})$. Since the series $[u]_{p^{k-1}}$ is not divisible by p, $Q(u)$ is divisible by $[u]_{p^{k-1}}$. Consequently one can divide both sides of the equation by $[u]_{p^{k-1}}$. Continuing the division we get $P(u) = \theta_p([u]_{p^{k-1}})Q_1(u)$.

We consider the $\mathbf{Z}_{p^k}$-space X_p, consisting of p points, on which a generator of $\mathbf{Z}_{p^k}$ acts by cyclic permutation. The vector $\mathbf{Z}_{p^k}$-bundle $X_p \times \Delta^n_1$ over X_p determines a bordism class which belongs to $\hat{\mathfrak{M}}^k_{2n}$. By what has already been shown,

$$\mathbf{D}\alpha(X_p \times \Delta^n_1) = \theta_p([u]_{p^{k-1}})\, \widetilde{Q}(u).$$

The inclusion of the subgroup $\mathbf{Z}_{p^{k-1}}$ in $\mathbf{Z}_{p^k}$ induces a map $i\colon B\mathbf{Z}_{p^{k-1}} \to B\mathbf{Z}_{p^k}$.

The proof of the following lemma is obtained by direct verification, using the construction of the transfer homomorphism t from [1].

Lemma. *On the sphere S^{2n-1} let the action of the group $\mathbf{Z}_{p^{k-1}}$ be obtained by restriction of the action Δ^N_1 of $\mathbf{Z}_{p^k}$. Then the diagram*

$$\begin{array}{ccc} U_{2m-1}(S^{2N-1}/\Delta^N_1) & \xrightarrow{t} & U_{2m-1}(S^{2N-1}/\mathbf{Z}_{p^{k-1}}) \\ \downarrow D_N & & \downarrow D'_N \\ U^{2(N-m)}(S^{2N-1}/\Delta^N_1) & \xrightarrow{i^*} & U^{2(N-m)}(S^{2N-1}/\mathbf{Z}_{p^{k-1}}) \end{array}$$

is commutative. D'_N is the duality isomorphism for the manifold $S^{2n-1}/\mathbf{Z}_{p^{k-1}}$.

Since on X_p the action of the subgroup $\mathbf{Z}_{p^{k-1}}$ is trivial,

$$i^*\mathbf{D}\alpha(X_p \times \Delta^n_1) = D'_N t(X_p \times \Delta^N_1) = p.$$

But $i^*\theta_p([u]_{p^{k-1}}) = p$, whence it follows that $\widetilde{Q}(u) = 1$ and $D\alpha(X_p \times \Delta_1^n) = \theta_p([u]_{p^{k-1}})$. The theorem is proved.

1.5. This subsection lies somewhat outside the basic goals of this paper; however, it is connected with the rest of the general methods of proof, which allow one to obtain a more geometrical interpretation of the "integrality" theorem in cobordism obtained in [8].

Let i be an imbedding of the G-manifold M in the space of a representation $\widetilde{\Delta}$; it induces an imbedding of the space $(M \times EG)/G$ in the fiber space of the vector bundle $\nu(\widetilde{\Delta})$ with complex normal bundle. The Thom construction gives the cobordism class of the Thom space of the bundle $\nu(\widetilde{\Delta})$, which will be denoted by $\lambda(M) \in U^*(M\nu(\widetilde{\Delta}))$. Let Φ: $U^*(M\nu(\widetilde{\Delta})) \to U^*(BG)$ be the Thom isomorphism; then there is a well-defined homomorphism $\mu: U_{2n}^G \to U^{-2n}(BG)$ whose value on M is equal to $\Phi\lambda(M)$.

If the action of G on M had no fixed points, one could assume that the imbedding i is an equivariant map into the sphere of the representation space. Just as in the previous subsection, we remark that $\lambda(M)$ under the homomorphism $U^*(M\nu(\widetilde{\Delta})) \to U^*(E\nu(\widetilde{\Delta}))$ is carried to zero. Since this homomorphism coincides with the composition of the isomorphism Φ and multiplication by $e(\widetilde{\Delta})$, we have proved

Theorem. *The homomorphism μ maps the submodule $\hat{U}_*^G$ into the ideal $I^*(G)$.*

1.6. As was shown in §1.4, the factor ring $U^*(BZ_{p^k})/I^*(Z_{p^k})$ is isomorphic to the ring $U^*[[u]]/(\theta_p([u]_{p^{k-1}}) = 0)$. Let $\overline{D\alpha}$ denote the composition of the homomorphism $D\alpha$ and the projection $U^*(BZ_{p^k}) \to U^*[[u]]/(\theta_p([u]_{p^{k-1}}) = 0)$. From Theorem 1.5 it follows that the homomorphism $\overline{D\alpha}$ can be represented in the form

$$\mathfrak{M}_*^k \longrightarrow \mathfrak{M}_*^k/\hat{\mathfrak{M}}_*^k \longrightarrow U^0(BZ_{p^k})/I^0\ (Z_{p^k}).$$

Recalling the remark to Lemma 1.3, we get

Corollary. *A coset of the factor module $\mathfrak{M}_{2n}^k/\hat{\mathfrak{M}}_{2n}^k$ belongs to* $\operatorname{Im}\delta$ *if and only if, for a representative $[\zeta] \in \mathfrak{M}_{2n}^k$ of this coset, $\overline{D\alpha}(\zeta)$ is divisible by u^n in the ring* $U^*[[u]]/(\theta_p([u]_{p^{k-1}}) = 0)$.

1.7. Let M be an n-dimensional manifold with an action of the group Z_{p^k}. The free action of Z_{p^k} by Δ_1^N on S^{2N-1} gives a free action on the product $M \times S^{2N-1}$. If ϕ_N is the projection, ϕ_N: $(M \times S^{2N-1})/Z_{p^k} \to S^{2N-1}/\Delta_1^N$, then

$$\varphi_N(M) = [(M \times S^{2N-1})/Z_{p^k}, \varphi_N].$$

It is easy to verify that there exists a cobordism class $D\phi(M) \in U^{-2n}(BZ_{p^k})$ such that $i_N^* D\phi(M) = D_N\phi_N(M)$.

In this subsection we shall assume that M is a singular manifold (this is equivalent to triviality of the action of Z_p).

Theorem. *Let* $[\zeta] = [M \times (\mathbf{CP}^{n_1}_{j_1}) \times \cdots \times (\mathbf{CP}^{n_r}_{j_r})]$ *be a bordism class belonging to* $\mathbf{M}^k_*$ *(from the definition of* $\mathbf{M}^k_*$ *it follows that* $(j_s, p) = 1$*). Then* $\mathrm{D}\alpha(\zeta)$ *satisfies the equation*

$$\prod_s e(\Delta_{j_s}^{n_s+1})\,\mathbf{D}\,\alpha(\zeta) = e(\Delta_1^n)\,\mathbf{D}\,\varphi(M)\cdot\prod_s e(\Delta_1^{n_s+1})\,\pi_!^s e(\eta_s\otimes\xi^{j_s}),$$

where η_s *is the bundle over* $\mathbf{CP}^{n_s}$ *such that its sum with the canonical bundle* ξ_s *over* $\mathbf{CP}^{n_s}$ *is trivial,* ξ *is the canonical bundle over* $\mathbf{BZ}_{p^k}$, *and* $\pi_!^s$ *is the Gysin homomorphism* [10] *corresponding to the map* $\pi^s\colon \mathbf{CP}^{n_s}\times\mathbf{BZ}_{p^k}\to\mathbf{BZ}_{p^k}$.

Proof. For any space X, the zero-dimensional bundle over it will be denoted by 1_X. The vector $\mathbf{Z}_{p^k}$-bundle ζ_1 by definition is equal to the sum of the vector $\mathbf{Z}_{p^k}$-bundles

$$\widetilde{\xi}_s = 1_M\times 1_{\mathbf{CP}^{n_1}}\times\cdots\times(\xi_s\otimes\Delta^1_{j_s})\times\cdots\times 1_{\mathbf{CP}^{n_r}}.$$

By analogy, for the vector bundle η_s we construct the vector $\mathbf{Z}_{p^k}$-bundle $\widetilde{\eta}$. The fiber space of the sphere bundle of the vector $\mathbf{Z}_{p^k}$-bundle $\zeta\oplus_s\widetilde{\eta}_s\oplus l$, which we denote by $F_l(\zeta)$, coincides with the $\mathbf{Z}_{p^k}$-manifold $M\times\mathbf{CP}^{n_1}\times\cdots\times\mathbf{CP}^{n_r}\times S^{2(n+l)-1}$, on whose sphere $S^{2(n+l)-1}$, $\Delta_1^l\oplus_s\Delta_{j_s}^{n_s+1}$ acts $(N=\Sigma_s(n_s+1))$.

By analogy with lemmas of [10] one proves

Lemma 1. *Let* $i\colon S(\zeta\oplus l)/\mathbf{Z}_{p^k}\to F_l(\zeta)/\mathbf{Z}_{p^k}$ *be the inclusion,* D_l^F *the duality isomorphism for the manifold* $F_l(\zeta)/\mathbf{Z}_{p^k}$, *and* $\widetilde{\eta}'_s$ *be the* $\mathbf{Z}_{p^k}$*-bundle over the* $\mathbf{Z}_{p^k}$*-manifold* $F_l(\zeta)$ *obtained from the bundle* $\widetilde{\eta}_s$ *by the map* $F_l(\zeta)\to M\times\mathbf{CP}^{n_1}\times\cdots\times\mathbf{CP}^{n_r}$. *Then*

$$\mathbf{D}_l^F[S(\zeta\oplus l)/\mathbf{Z}_{p^k}, i] = e(\bigoplus_s\widetilde{\eta}'_s/\mathbf{Z}_{p^k}).$$

We consider the $\mathbf{Z}_{p^k}$-equivariant map which is the identity on the first $r+1$ factors, $F_l(\zeta)$ on the $\mathbf{Z}_{p^k}$-manifold $M\times\mathbf{CP}^{n_1}\times\cdots\times\mathbf{CP}^{n_r}\times S^{2(N+l)-1}$, whose action on $S^{2(N+l)-1}$ is Δ_1^{N+l}. The corresponding map of factor spaces we denote by f_2. Repeating almost verbatim the proof of Theorem 1 of [5], we get the following lemma.

Lemma 2. *If*

$$\pi_{N+l}\colon(M\times\cdots\times S^{2(N+l)-1})/\mathbf{Z}_{p^k}\to S^{2(N+l)-1}/\Delta_1^{N+l},$$

then

$$\left[\prod_s\pi^*_{N+l}e(\Delta_{j_s}^{n_s+1})\right]\cdot f_{2!}(1) = \prod_s\pi^*_{N+l}e(\Delta_1^{n_s+1}).$$

The vector $\mathbf{Z}_{p^k}$-bundle over $M\times\cdots\times\mathbf{CP}^{n_r}\times S^{2(N+l)-1}$ obtained from $\widetilde{\eta}_s$ by the projection $M \times \cdots \times S^{2(N+l)-1} \to M \times \cdots \times \mathbf{CP}^{n_r}$ we denote by $\tilde{\eta}_s''$. From the definition of f_2 it follows that $\tilde{\eta}_s'/\mathbf{Z}_{p^k} = f_2^*(\tilde{\eta}_s''/\mathbf{Z}_{p^k})$. Further, $\tilde{\mathbf{D}}_{N+l}$ is the duality isomorphism for the manifold $(M \times \cdots \times S^{2(N+l)-1})/\mathbf{Z}_{p^k}$. To the equation proved in Lemma 1 of this subsection we apply the homomorphism f_2:

$$\tilde{\mathbf{D}}_{N+l}[S(\zeta \oplus l)/\mathbf{Z}_{p^k}, f_2 \circ i] = f_{2!}(f_2^* e(\oplus \tilde{\eta}_s''/\mathbf{Z}_{p^k})) = f_{2!}(1)\, e(\oplus \tilde{\eta}_s'/\mathbf{Z}_{p^k}).$$

Then

$$\prod_s \pi_{N+l}^* e(\Delta_{j_s}^{n_s+1})\, \tilde{D}_{N+l}[S(\zeta \oplus l)/\mathbf{Z}_{p^k}, f_2 \circ i] = \prod_s \pi_{N+l}^* e(\Delta_1^{n_s+1})\, e(\tilde{\eta}_s''/\mathbf{Z}_{p^k}).$$

By construction we have $[S(\zeta \oplus l)/\mathbf{Z}_{p^k}, \pi_{N+l} \circ f_2 \circ i] = \tilde{\alpha}(\zeta \oplus l)$ if we identify $S^{2(N+l)-1}/\Delta_1^{N+l}$ with the skeleton of $S^{2(N+l+n)-1}/\Delta_1^{n+l+N}$. Then

$$\mathbf{D}_{N+l+n}\tilde{\alpha}(\zeta \oplus l) = e(\Delta_1^n)\, D_{N+l}[S(\zeta \oplus l)/Z_{p^k}, \pi_{N+l} \circ f_2 \circ i].$$

We apply the homomorphism $\pi_{N+l!}$ to the preceding equation:

$$\prod_s e(\Delta_{j_s}^{n_s+1})\, \mathbf{D}_{N+l+n}\tilde{\alpha}(\zeta \oplus l) = e(\Delta_1^n)\, \pi_{N+l!} e(\oplus \tilde{\eta}_s''/\mathbf{Z}_{p^k}) \prod_s e(\Delta_1^{n_s+1}).$$

We represent the projection π_{N+l} in the form of the composition $\pi_{N+l}'' \circ \pi_{N+l}'$,

$$\pi_{N+l}' : (M \times \ldots \times S^{2(N+l)-1})/\mathbf{Z}_{p^k} \to \mathbf{CP}^{n_1} \times \ldots \times \mathbf{CP}^{n_r} \times S^{2(N+l)-1}/\Delta_1^{N+l},$$
$$\pi_{N+l}'' : \mathbf{CP}^{n_1} \times \ldots \times \mathbf{CP}^{n_r} \times S^{2(N+l)-1}/\Delta_1^{N+l} \to S^{2(N+l)}/\Delta_1^{N+l}.$$

If $\tilde{\eta}_s'''$ is the vector bundle over $\mathbf{CP}^{n_1} \times \cdots \times \mathbf{CP}^{n_r} \times \mathbf{BZ}_{p^k}$ equal to

$$1_{\mathbf{CP}^{n_1}} \times \ldots \times (\eta_s \otimes \xi^{j_s}) \times \ldots \times 1_{\mathbf{CP}^{n_r}},$$

then $\tilde{\eta}_s''/\mathbf{Z}_{p^k} = \pi_{N+l}'^*(\tilde{\eta}_s''')$. Then

$$\pi_{N+l!}'(e(\oplus \tilde{\eta}_s''/\mathbf{Z}_{p^k})) = e(\oplus \tilde{\eta}_s''')\, \pi_{N+l!}'(1).$$

From the definition of the homomorphism $\mathbf{D}\phi$ it follows that

$$\pi_{N+l!}'(1) = \pi_{N+l}''^* \mathbf{D}\varphi(M).$$

To complete the proof it remains to note that

$$\pi_{N+l!}'' e(\oplus \tilde{\eta}_s''') = \prod \pi_!^s e(\eta_s \otimes \xi^{j_s}).$$

1.8. Since multiplication by the Euler class of a representation in the ring $U^*[[u]]/(\theta_p([u]_p^{k-1}) = 0)$ is a monomorphism, equations which are satisfied by $\mathbf{D}\alpha(\zeta)$ are solvable for $\mathbf{D}\alpha(\zeta)$.

Corollary. *Let $A_n(v, u)$ be the series defined by $A_n(v, u) f(v, u) = 1 \otimes u^{n+1}$ in the* *ring* $U^*[[v, u]]/v^{n+1} = 0$, *where* $f(v, u)$ *is the formal group of "geometric" cobordism. The sequence* $B_n(u)$ *is obtained from* $A_n(v, u)$ *by replacing* v^k *by* $[\mathbf{CP}^{n-k}]$ $(k = 0, 1, 2, \cdots)$. *For the vector* $\mathbf{Z}_{p^k}$*-bundle* $\zeta = (\mathbf{CP}^{n_1}_{j_1}) \times \cdots \times (\mathbf{CP}^{n_r}_{j_r})$, $(j_s, p) = 1$,

$$\overline{\mathbf{D}\alpha}(\zeta) = \prod_s \frac{e(\Delta_1^{n_s+1})}{e(\Delta_{j_s}^{n_s+1})} B_{n_s}(e(\Delta^1_{j_s})).$$

Remark. From the definition of the sequence $B_n(u)$ it is simple to verify that

$$\left(\sum_{n=0}^{\infty} B_n(u)\, t^n\right) f(u, ut) = u\left(\sum_{n=0}^{\infty} [\mathbf{CP}^n]\, u^n t^n\right).$$

1.9. Since by definition of the module $R_i^{\mathbf{Z}_{p^k}}$ the restriction of the action of the group $\mathbf{Z}_{p^k}$ to the sphere bundle of the vector $\mathbf{Z}_{p^k}$-bundle ζ, $[\zeta] \in R_{2n}^{\mathbf{Z}_{p^k}}$, has no fixed points, there exists a continuous $\mathbf{Z}_{p^k}$-equivariant map of $\mathbf{S}\zeta$ into the sphere S^{2n-1} with the action of $\Delta^n_{p^k-1}$ (cf. §1.4). We extend it by linearity to a continuous $\mathbf{Z}_{p^k}$-equivariant map of $\mathbf{E}\zeta$ into the space C^n of the representation $\Delta^n_{p^k-1} g_1 \colon \mathbf{E}\zeta \to C^n$. If ψ_N is a map gotten by smoothing the continuous factor map

$$(g_1 \times \mathrm{id})/\mathbf{Z}_{p^k} \colon (\mathbf{E}\zeta \times S^{2N-1})/\mathbf{Z}_{p^k} \to (C^n \times S^{2N-1})/\mathbf{Z}_{p^k}$$

(the action on the sphere S^{2N-1} is Δ_1^N), then it determines the corresponding bordism class

$$\psi_N(\zeta) = [(\mathbf{E}\zeta \times S^{2N-1})/\mathbf{Z}_{p^k}, \quad (\mathbf{S}\zeta \times S^{2N-1}/\mathbf{Z}_{p^k});\ \psi_N] \in U_{2(N+n)-1}(\mathbf{E}\rho_n, \mathbf{S}\rho_n).$$

There exists a cobordism class $\mathbf{D}\psi(\zeta) \in U^0(\mathbf{BZ}_{p^k})$ such that $i_N^* \mathbf{D}\psi(\zeta) = s_0^* \mathbf{D}'(\psi_N(\zeta))$. Thus we have constructed the homomorphism $\mathbf{D}\psi \colon R_*^{\mathbf{Z}_{p^k}} \to U^0(\mathbf{BZ}_{p^k})$.

Lemma. *For an arbitrary* $\mathbf{Z}_{p^k}$*-manifold* M,

$$e(\Delta^n_{p^k-1})\,\mathbf{D}\varphi(M) = \mathbf{D}\psi(\beta^{\mathbf{Z}_{p^k}}(M)).$$

Proof. The complement of a tubular neighborhood of the fixed submanifold of the action of $\mathbf{Z}_{p^k}$ on the manifold M is mapped continuously and $\mathbf{Z}_{p^k}$-equivariantly into the sphere S^{2N-1} with the action $\Delta^n_{p^k-1}$. Continuing this map by linearity onto the tubular neighborhood, we get a continous $\mathbf{Z}_{p^k}$-equivariant map $g_2 \colon M \to C^n$. The map which is a smoothing of

$$(g_2 \times \mathrm{id})/\mathbf{Z}_{p^k} \colon (M \times S^{2N-1})/\mathbf{Z}_{p^k} \to (C^n \times S^{2N-1})/\mathbf{Z}_{p^k},$$

determines a bordism class which obviously coincides with $s_{0*}\,\phi_N(M) \in U_{2(N+n)-1}(\mathbf{E}\rho_n)$. The assertion of the lemma follows from the definition of the homomorphism $\mathbf{D}\psi$ and the diagram of §1.4.

1.10. For the homomorphism $D\psi$ one has the analogue of Theorem 1 of [5] and § 1.7 of the present paper.

Theorem 1. *Let* Δ *be an n-dimensional representation of the group* $\mathbf{Z}_{p^k}$ *which has no trivial summand. Then* $e(\Delta)\cdot D\,\psi(\Delta) = e(\Delta^n_{p^{k-1}})$.

Proof. We consider the inclusion $i: E(\rho_n) \subset E(\rho_n \oplus \nu(\Delta))$. Obviously the cobordism class dual to $i_*\psi_N(\Delta)$ is equal to $i_N^*(D\psi(\Delta)\cdot e(\Delta))$. We are using the fact that the two $\mathbf{Z}_{p^k}$-equivariant maps h_1 and h_2 of the space of the representation Δ into the space of the representation $\Delta + \Delta^n_{p^{k-1}}$ are $\mathbf{Z}_{p^k}$-homotopic. This means that the bordism class $i_*\psi_N(\Delta)$ coincides with the bordism class given by the inclusion $E(\nu(\Delta)) \to E(\rho_n \oplus \nu(\Delta))$. But here the normal bundle to the image of the inclusion is ρ_n, and this means that the dual cobordism class is equal to $e(\Delta^n_{p^{k-1}})$. Thus we have obtained the equation we are proving.

By an insignificant change in the proof of Theorem 1.7, we get the following theorem.

Theorem 2. *The value of the homomorphism* $D\psi$ *on the vector* $\mathbf{Z}_{p^k}$*-bundle* $\zeta = (\mathbf{CP}^{n_1}_{j_1}) \times \cdots \times (\mathbf{CP}^{n_r}_{j_r})$ *satisfies the equation*

$$\prod_s e(\Delta^{n_s+1}_{j_s})\, D\,\psi(\zeta) = \prod_s e(\Delta^{n_s+1}_{p^{k-1}})\, \pi^s_! e(\eta_s \otimes \xi^{j_s}).$$

Remark. Solving the equation in the ring $U^*[[u]]/(\theta_p([u]_{p^{k-1}}) = 0)$, one can write for the class $D\psi(\zeta)$

$$\overline{D\,\psi}(\zeta) = \prod_s \frac{e(\Delta^{n_s+1}_{p^{k-1}})}{e(\Delta^{n_s+1}_{j_s})} B_{n_s}(e(\Delta^1_{j_s})).$$

1.11. In this subsection we shall sum up all the results obtained above. We assume that one has already gotten the description of $\operatorname{Im}\beta^{\mathbf{Z}_{p^k}}$ for $l < k$.

The vector $\mathbf{Z}_{p^k}$-bundle ζ, whose bordism class $[\zeta] \in R^{\mathbf{Z}_{p^k}}_{2n}$, we represent uniquely in the form

$$\sum_l \left(\sum_m a_{m,l}\zeta_{m,l}\right)\zeta_l,$$

where $\zeta_{m,l}$ and ζ_l are monomials of the form $(\mathbf{CP}^{n_1}_{j_1}) \times \cdots \times (\mathbf{CP}^{n_r}_{j_r})$, for which $(j_s, p) = p$ and $(j_s, p) = 1$, respectively, and $a_{m,l} \in U_*$. The subgroup $\mathbf{Z}_p$ acts trivially on the fiber space of the bundle $\zeta_{m,l}$, so it can be considered as a $\mathbf{Z}_{p^{k-1}}$-bundle ($\mathbf{Z}_{p^{k-1}} = \mathbf{Z}_{p^k}/\mathbf{Z}_p$).

Theorem. *The bordism class*

$$\sum_l \left(\sum_m a_{m,l}\zeta_{m,l}\right)\zeta_l \in R_{2n}^{\mathbf{Z}_{p^k}} (\dim \zeta_l = 2n_l)$$

belongs to the image of the homomorphism $\beta^{\mathbf{Z}_{p^k}}$ *if and only if*

1) *For any* l *the sum* $\Sigma_m a_{m,l}\zeta_{m,l}$, *considered as a bordism class belonging to* $R_{2(n-n_l)}^{\mathbf{Z}_{p^{k-1}}}$, *lies in* $\operatorname{Im}\beta^{\mathbf{Z}_{p^{k-1}}}$,

2) $$\sum_l \frac{e(\Delta_1^{n-n_l}) \sum_m a_{m,l}\overline{\mathbf{D}\psi}(\zeta_{m,l})}{e(\Delta_{p^{k-1}}^{n-n_l})} \overline{\mathbf{D}\alpha}(\zeta_l)$$

is divisible by u^n *in the ring* $U^*[[u]]/(\theta_p([u]_{p^{k-1}}) = 0)$.

(The values of the homomorphisms $\overline{\mathbf{D}\psi}(\zeta_{m,l})$ and $\overline{\mathbf{D}\alpha}(\zeta_l)$ are given by the remark to Theorem 2 of the preceding subsection and Corollary 1.8.)

§2. Admissible collections of fixed submanifolds of the action of a cyclic group of finite order

The possibility of reducing the problem of admissible collections of fixed submanifolds of the action of the group $\mathbf{Z}_m$ to the analogous problem for its p-primary components was indicated to the author by S. M. Guseĭn-Zade.

2.1. We assume that for any cyclic group $\mathbf{Z}_{m_1}$ of order less than m we have already obtained the description of $\operatorname{Im}\beta^{\mathbf{Z}_{m_1}}$. From the results of §1 it follows that without loss of generality one can assume that $m = p_1^{k_1} \times \cdots \times p_r^{k_r}$, $r > 1$. Analogous to the module $\hat{\mathfrak{M}}_*^k$, we define the module $\hat{\mathfrak{M}}_*^{\mathbf{Z}_m}$ as the module of vector $\mathbf{Z}_m$-bundles on whose sphere bundle the group acts freely and on whose fiber space there are no fixed points.

Lemma. *The homomorphism* $\alpha\colon \hat{\mathfrak{M}}_{2n}^{\mathbf{Z}_m} \to \tilde{U}_{2n-1}(B\mathbf{Z}_m)$ *is an epimorphism.*

Remark. Wherever the contrary is not asserted, the definitions and notation are automatically carried over from §1.

Proof. The group $\mathbf{Z}_m$ is isomorphic to the direct sum $\mathbf{Z}_{p_1^{k_1}} \oplus \cdots \oplus \mathbf{Z}_{p_r^{k_r}}$. We denote by X_{p_i}, $i = 1, 2$, the $\mathbf{Z}_m$-space consisting of p_i points, on which a generator of the group $\mathbf{Z}_{p_i^{k_i}}$ acts by cyclic permutation, and the remaining generators act trivially. Just as in the proof of epimorphicity in Theorem 1.4, we get that

$$\mathbf{D}\alpha(X_{p_i} \times \Delta_1^n) = \theta_{p_i}\left([u]_{\frac{m}{p_i}}\right).$$

Since the free term in the series $\theta_{p_i}(u)$ is equal to p_i, and $(p_1, p_2) = 1$, the ideal of the ring $U^*(BZ_m)$ generated by the series $\theta_{p_i}([u]_{m/p_i})$ coincides with the whole ring. This means that the homomorphism $D\alpha$ is an epimorphism. This implies that the homomorphism α is an epimorphism.

2.2. For any $1 \le s \le r$ we represent the bordism class $r \in R_*^{Z_m}$ in the form $\Sigma a^s_{m,n} \zeta^s_{m,n} \zeta^s_n$, where $\zeta^s_{m,n}$ and ζ^s_n are monomials $(CP^{n_1}_{j_1}) \times \cdots \times (CP^{n_l}_{j_l})$ for which $(j_i, p_s) = p_s$ and $(j_i, p_s) = 1$ respectively. The subgroup Z_{p_s} acts trivially on the fiber space of the bundle $\zeta^s_{m,n}$; hence it can be considered as a Z_{m/p_s}-bundle. From the lemma of the preceding subsection it is easy to get

Theorem. *The bordism class $r \in R_*^{Z_m}$ belongs to $\operatorname{Im} \beta^{Z_m}$ if and only if for any s and n the sum $\Sigma_m a^s_{m,n} \zeta^s_{m,n}$, considered as a bordism class in $R_*^{Z_{m/p_s}}$, belongs to* $\operatorname{Im} \beta^{Z_{m/p_s}}$.

§3. Manifolds which realize admissible collections of fixed submanifolds

3.1. The kernel of the homomorphism β^G is the submodule of $\hat{U}^G_*$ of bordism classes of manifolds on which the action of the group has no fixed points. Hence the problem of reconstructing the bordism class of a manifold by fixed invariants has a solution only modulo $\operatorname{Ker} \pi$, where $\pi: U^G_* \to U_*$ is the homomorphism "forgetting" the action of G on the manifold.

We shall show that $\pi \hat{U}^{Z_m} = U_*$ for $m = p_1^{k_1} \times \cdots \times p_r^{k_r}$, $r > 1$. In fact, there exist integers a and b such that $ap_1 + bp_2 = 1$. Recalling the definition of the spaces X_{p_1} and X_{p_2}, we get that

$$[M \times (aX_{p_1} + bX_{p_2})] \in \hat{U}_*^{Z_m} \quad \text{and} \quad \pi [M \times (aX_{p_1} + bX_{p_2})] = [M]$$

for any $[M] \in U_*$.

3.2. We consider any n-dimensional manifold M with an action of the group Z_{p^k}, and let the normal bundle to the singular submanifold of the action give the bordism class $[\zeta]$, belonging to $\mathfrak{M}^k_{2n}$. From Theorem 35.2 of [1] it follows that $\zeta \oplus 1 - [M]\Delta^1_1$ belongs to $\operatorname{Ker} \alpha$. This means $D\alpha(\zeta) - [M]u^n$ is divisible in the ring $U^*[[u]]/([u]_{p^k} = 0)$ by u^{n+1}.

Corollary. *If on the manifold M the group Z_{p^k} acts without fixed points, then the bordism class of M is divisible by p.*

Proof. From Theorem 1.4 it follows that $D\alpha(\zeta)$ lies in the ideal generated by the series $\theta_p([u]_{p^{k-1}})$. Since the free term of this series is equal to p, the corollary is proved.

Thus we have obtained the fact that $\pi\, \hat{U}_*^{\mathbf{Z}_{p^k}}$ is isomorphic to pU_*.

We identify U_* with its image under the imbedding in $U_* \otimes Q$, and we then have

Theorem. *There is defined a homomorphism γ_p^k from the module $R_*^{\mathbf{Z}_{p^k}}$, taking values in $U_* \otimes Q$, such that, for any manifold which realizes an admissible collection of $\mathbf{Z}_{p^k}$-bundles r,*

$$[M] = \gamma_p^k(r) \pmod{pU_*}.$$

The value of γ_p^k on an arbitrary collection $r = \Sigma a_{m,l}\zeta_{m,l}\zeta_l$ (notation of §1.11) is given by the formula

$$\gamma_p^k(r) = \left[\frac{p}{\vartheta_p([u]_{p^{k-1}})} \sum_l \frac{e(\Delta_1^{n-n_l}) \sum_m a_{m,l} \widetilde{\mathbf{D}\psi}(\zeta_{m,l})}{e(\Delta_{p^{k-1}}^{n-n_l})} \widetilde{\mathbf{D}\alpha}(\zeta_l) \right]_n ,$$

where the values of the homomorphisms $\widetilde{\mathbf{D}\psi}$ and $\widetilde{\mathbf{D}\alpha}$ are given by formulas which coincide with the formulas for $\overline{\mathbf{D}\psi}$ and $\overline{\mathbf{D}\alpha}$. (It is necessary to remark that division in these formulas for the homomorphisms $\widetilde{\mathbf{D}\psi}$ and $\widetilde{\mathbf{D}\alpha}$ must be carried out in the ring $U_*[[u]] \otimes Q$.)

Remark. An analogous theorem for the group $\mathbf{Z}_p$ was obtained in [11].

Proof. Let $P_1(u) \in U^*[[u]]$ represent $\overline{\mathbf{D}\alpha}(\zeta)$. Then

$$[M]u^n = P_1(u) + \theta_p([u]_{p^{k-1}})Q_1(u) + u^{n+1}Q_2(u).$$

Since the free term of the series $p/\theta_p([u]_{p^{k-1}})$ is equal to 1, if we multiply both sides of the preceding equation by it, we get

$$[M]u^n = \frac{p}{\theta_p([u]_{p^{k-1}})} P_1(u) + pQ_1(u) + u^{n+1}Q_2'(u).$$

From Theorem 1.11 and the fact that the difference of the values of the homomorphisms $\overline{\mathbf{D}\psi}$ and $\widetilde{\mathbf{D}\psi}$, and also of $\overline{\mathbf{D}\alpha}$ and $\widetilde{\mathbf{D}\alpha}$, lies in the ideal generated by $\theta_p([u]_{p^{k-1}})$, the assertion we are proving follows.

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