A Conference on Integrable Systems in Algebra, Geometry, and Physics

Conference Program

Wednesday, May 4 (Faculty House, Presidential Rooms 2 and 3)

9:00–9:30	Breakfast	
9:30–10:15	*S. Novikov*	**Supersymmetric 2D Pauli Operators and Riemann Surfaces**
10:30–11:15	*M. Ablowitz*	**Chazy-Ramanujan Type Equations**
11:30-12:15	*P. Deift*	**On the Initial Boundary Problem for the NLS Equation**
12:30–2:30	Break	
2:30–3:15	*V. Zakharov*	**Spaces of Diagonal Curvature and Orthogonal Coordinate Systems**
3:15–4:00	Coffee	
4:00–4:45	*V. Buchstaber*	**Krichever's Formal Groups and Deformed Baker-Akhiezer Functions**

Thursday, May 5 (Faculty House, Presidential Rooms 2 and 3)

9:00–9:30	Breakfast	
9:30–10:15	*G. Olshanski*	**Integrable Kernels and Markov Dynamics**
10:30–11:15	*P. Etingof*	**Elliptic Calogero-Moser Systems for Crystallographic Complex Reflection Groups**
11:30-12:15	*A. Vershik*	**Fock Space as Integral over Trajectories (Configurations) of the Random Field and Non-Fock Factorizations**
12:30–2:30	Break	
2:30–3:15	*A. Its*	**Quantum Entanglement and the Finite Gap Integration**
3:15–4:00	Coffee	

Friday, May 6 (Faculty House, Presidential Rooms 2 and 3)

9:00–9:30	Breakfast	
9:30–10:15	*P. Wiegmann*	**Emergent Conformal Symmetry in Selberg-Dyson's Integrals**
10:30–11:15	*N. Nekrasov*	**Quantum Integrability and Gauge Theories**
11:30-12:15	*A. Veselov*	**Gaudin Subalgebras and Moduli Space of Stable Rational Curves**
12:30–2:30	Break	
2:30–3:15	*A. Marshakov*	**Lie Groups, Toda Chains and Cluster Variables**
3:15–4:00	Coffee	
4:00–4:45	*O. Babelon*	**Moment Map and Bethe Ansatz in the Jaynes-Cummings-Gaudin Model**
6:30–10:30	**Dinner**	*Faculty House Skyline Dining Room*

Saturday, May 7 (Mathematics 312)

9:00–9:30	Breakfast	*Mathematics Common Room*
9:30–10:15	*Y. Sinai*	**Elementary Estimates of the Riemann Zeta-Function**
10:30–11:15	*L. Takhtajan*	**On Opers and Liouville Equation**
11:30-12:15	*T. Shiota*	**Abelian Solutions to Soliton Equations**
12:30–2:30	Break	
2:30–3:15	*A. Braverman*	**Quantum K-theory of Affine Flag Manifolds, q-deformed Toda Equations and Uhlenbeck Spaces**
3:15–4:00	Coffee	
4:00–4:45	*A. Borodin*	**Tau-function of Discrete Isomonodromy Transformations and Probability**
5:00	**Reception**	*Mathematics Common Room*

Abstracts

(1) *Mark Ablowitz* (University of Colorado at Boulder, USA)

Title: Chazy-Ramanujan Type Equations

Abstract: At the turn of the last century Painlevé and his school obtained a class of second order ordinary differential equations (ODE's) in the complex ("time") plane which have no movable branch points. The ODEs they found include equations with elliptic function solutions and the now well known six Painlevé equations. Chazy (1909-11) studied third order ODEs and found equations whose solutions are analytic inside/outside a circle, which consists of a movable natural boundary. Chazy's equations are intimately connected to a system of equations studied by Darboux-Halphen (1878) and a generalized 9^{th} order system obtained by reduction from the self-dual Yang-Mills equations. The general solution of Chazy's equation can be written, compactly in terms of modular forms associated with the modular group and can be related to Eisenstein series. Remarkably, Chazy's equation also corresponds to a system of equations studied by Ramanujan in 1916 describing certain Eisenstein series. Recently it has been shown that different third order nonlinear ODEs 'of Chazy-type' can be associated with subgroups of the modular group and their Eisenstein series. If time permits water waves, integrability and line soliton solutions will be mentioned.

(2) *Olivier Babelon* (LPTHE - CNRS - Université Paris 6, France)

Title: Moment Map and Bethe Ansatz in the Jaynes-Cummings-Gaudin Model

Abstract: We study the image of the moment map in the JCG model. Singularities are of the elliptic and focus-focus type. The determination of normal forms around such singularities is very easily achieved by a classical analog of Algebraic Bethe Ansatz. Focus-focus singularities are obstructions to the existence of global action-angle coordinates. We will see their consequences in the quantum case and how Bethe equations seem to "know" about them.

(3) *Victor Buchstaber* (Steklov Mathematical Institute, Russia)

Title: Krichever's Formal Groups and Deformed Baker-Akhiezer Functions

Abstract: We describe explicitly the formal group law over $\mathbb{Z}[\mu]$ corresponding to the Tate uniformization of the general Weierstrass model of the cubic curve with parameters $\mu = (\mu_1, \mu_2, \mu_3, \mu_4, \mu_6)$.This formal group law is called the general elliptic formal group law.

We introduce the universal Krichever formal group law with the ring of coefficients $\mathcal{A}_{\mathrm{Kr}}$. Its exponential is defined by the Baker-Akhiezer function $\Phi(t) = \Phi(t; \tau, g_2, g_3)$, where τ is a point on the elliptic curve with Weierstrass parameters (g_2, g_3). Results on the ring $\mathcal{A}_{\mathrm{Kr}}$ are obtained. We find the conditions necessary and sufficient for the elliptic formal group law to be a Krichever formal group law, and thus to define a rigid Hirzebruch genus on S^1-equivariant SU-manifolds.

We introduce the deformed Baker-Akhiezer function $\Psi(t) = \Psi(t; v, w, \mu)$. It is a quasi-periodic function with logarithmic derivative determined by the exponential of the general elliptic formal group law. Here v and w are points of the curve with parameters μ. The deformation parameter is $\alpha = \wp'(w)/\wp'(v)$, for $\alpha = \pm 1$ the function $\Psi(t)$ coincides with the Baker-Akhiezer function $\Phi(t)$. We obtain the addition theorem for $\Psi(t)$ and using this theorem we prove that the deformed Baker-Akhiezer function is the common eigenfunction of two double periodic differential operators of degrees 2 and 3. Their commutator has $(1 - \alpha^2)$ as a factor.

Main definitions will be introduced during the talk. New results presented in the talk have been obtained in recent joint works with E. Yu. Bunkova.

(4) *Percy Deift* (Courant Institute of Mathematical Sciences, New York University, USA)

Title: On the Initial Boundary Problem for the NLS Equation

Abstract: This is joint work with Jungwoon Park. We show how to obtain the long-time behavior of solutions of the IBV for NLS using the method of Bikbaev and Tarasov.

(5) *Pavel Etingof* (Massachusetts Institute of Technology, USA)

Title: Elliptic Calogero-Moser Systems for Crystallographic Complex Reflection Groups

Abstract: Let X be a complex abelian variety, and W a finite (complex) reflection group acting on X (such reflection groups are called crystallographic). In this case, one can define a family of classical or quantum integrable systems on X whose symbols are the W-invariant polynomials on the tangent space to X at the origin. These systems are parametrized by functions on the set of conjugacy classes of reflections in the double affine reflection group $W \ltimes L$, where L is the fundamental group of X. If $X = E^n$, where E is an elliptic curve, and $W = S_n$, these are the elliptic Calogero-Moser systems, and if $X = E^n$ and $W = S_n \ltimes \mathbb{Z}_2^n$, these are the Inozemtsev systems. However, when E is an elliptic curve with symmetries, $X = E^n$, and $W = S_n \ltimes \mathbb{Z}_\ell^n$, where $\ell = 3, 4, 6$, one gets new systems (with Hamiltonian of higher order than 2). I will explain how these (classical) systems arise in the study of finite-gap differential operators (of higher order) on elliptic curves with symmetries, similarly to how usual elliptic Calogero-Moser systems arise in the study of finite-gap Schrödinger oprators on elliptic curves, through the work of Airault-McKean-Moser and Krichever. This is based on joint work with Felder-Ma-Veselov and with Rains.

(6) *Alexander Veselov* (Loughborough University, UK)

Title: Gaudin Subalgebras and Moduli Space of Stable Rational Curves

Abstract: Gaudin subalgebras are abelian Lie subalgebras of maximal dimension spanned by the generators of the Kohno-Drinfeld Lie algebra $\mathfrak{t}_n$, which can be interpreted as the set of values for the universal Knizhnik-Zamolodchikov connection.

It turns out that Gaudin subalgebras form a smooth algebraic subvariety of the Grassmannian variety $Gr(n-1, n(n-1)/2)$ that is isomorphic to the moduli space of stable genus zero curves with $n+1$ marked points. I will discuss this and the relations with integrable systems and representation theory. The talk is based on a recent joint work with Aguirre and Felder.

(7) *Paul Wiegmann* (University of Chicago, USA)

Title: Emergent Conformal Symmetry in Selberg-Dyson's Integrals

Abstract: Dyson-Selberg integrals in the limit of large number of variables show emergent conformal symmetry. They transformed covariantly with respect to conformal transformations under a deformation of a contour of integration. A sequence of Dyson-Selberg integrals can be treated as a finite dimensional approximation of conformal blocks.