Algebraic Geometry, Mathematical Physics, and Solitons

Conference Program

(Times slots are in the Eastern Time Zone; UTC-4)

Friday, October 7

9:30-10:15	Coffee	
10:15–11:00	*Enrico Arbarello*	**KP equation, flexes, and trisecants to the Kummer variety**
11:15–12:00	*Alexander Veselov*	**Theta divisors and Chern-Dold character in complex cobordisms**
12:00–2:00	Lunch	
2:00–2:45	*Dmitry Zakharov*	**The tropical Prym variety**
2:45–3:30	Coffee	
3:30–4:15	*Tamara Grava*	**The Stieltjes-Fekete problem and degenerate orthogonal polynomials**
4:30–5:15	*Paul Wiegmann*	**Chiral anomalies in hydrodynamics**

Saturday, October 8

8:30–9:00	Coffee	
9:00–9:45	*Alexander Its*	**TT*-equations of Cecotti and Vafa, Riemann-Hilbert method and Iwasawa factorization**
9:45-10:15	Coffee	
10:15–11:00	*Pavel Etingof*	**Analytic Langlands correspondence over $\mathbb{C}$ and $\mathbb{R}$**
11:15–12:00	*Ivan Cherednik*	**L-functions and motivic superpolynomials of plane curve singularities**
12:00–2:00	Lunch	
2:00–2:45	*Alexander Varchenko*	**Polynomial solutions of KZ equations modulo an integer**
3:00–3:45	*Nikita Nekrasov*	**Integrable systems in gauge theories and sigma models**
3:45–4:15	Coffee	
4:15–5:00	*Andrei Okounkov*	**Equivariant cobordism and Eisenstein series**
6:00–9:00	Banquet	

Sunday, October 9

8:30–9:00	Coffee	
9:00–9:45	*Alisa Knizel*	**Discrete Painlevé equations and discrete log-gases**
9:45-10:15	Coffee	
10:15–11:00	*Alexander Braverman*	**Hecke operators on moduli spaces on bundles over local fields**
11:15–12:00	*Duong Phong*	**Geometric flows in symplectic geometry**
12:00–2:00	Lunch	
2:00–2:45	*Leon Takhtajan*	**On algebraic de Rham theorem**

Abstracts

(1) *Enrico Arbarello* (Accademia Nazionale dei Lincei, Rome, Italy)

Title: KP equation, flexes, and trisecants to the Kummer variety

Abstract: We describe a purely algebro-geometric way to achieve some of the beautiful and fundamental results obtained by *Igor Krichever* (and in one instance by *T. Shiota*) in characterizing Jacobians among principally polarized abelian varieties. This is based on some old work I did with *C. De Concini* and on recent work with *C. Codogni* and *G. Pareschi*.

(2) *Alexei Borodin* (Massachusetts Institute of Technology, Cambridge, MA, USA)

Title: $sl(1|1)$-vertex models: boson-fermion correspondence and determinantal point processes

Abstract: We consider probability measures on lattice paths defined via partition functions of a fully inhomogeneous free fermion six vertex model. These measures turn out to be determinantal with an explicit correlation kernel, and the proof is based on inhomogeneous versions of fermionic operators in a Fock space that originate from the algebraic Bethe Ansatz for the six vertex model.

(3) *Alexander Braverman* (University of Toronto, Toronto, Canada)

Title: Hecke operators on moduli spaces on bundles over local fields

Abstract: This talk is based on a joint work in progress with *D. Kazhdan* and *A. Polishchuk* and it will have some overlap with *P. Etingof*'s talk. Let $\mathbb{F}$ be a local field (archimedean or non-archimedean) and let C be a smooth projective curve over $\mathbb{F}$ of genus > 1. For a reductive group G we consider the moduli stack $\mathrm{Bun}_G(C)$ of principal G-bundles on C. We explain several approaches to defining some "nice" space of half-forms on $\mathrm{Bun}_G(C)$, and how some properties of this space are related to questions about algebraic geometry of G-bundles (we shall mostly concentrate on the case $G = \mathrm{SL}(2)$). In particular, our space will be acted on by a very large commutative algebra of Hecke operators, so one can look for Hecke eigen-forms. In the case when $\mathbb{F}$ is non-archimedian and C is regular over $\mathcal{O}_{\mathbb{F}}$ (the ring of integers of $\mathbb{F}$), we explain how to attach such an eigen-form to a cuspidal Hecke eigen-function defined over the corresponding finite (the latter is a classical object of the theory of automorphic forms).

(4) *Ivan Cherednik* (University of North Carolina, Chapel Hill, NC, USA)

Title: L-functions and motivic superpolynomials of plane curve singularities

Abstract: We will begin with a mini-review of the 4 main classical theories of zeta-functions and will try to motivate the passage to isolated singularities. Next, the motivic superpolynomials of plane curve singularities will be defined, which conjecturally coincide with the DAHA and Khovanov-Rozansky ones

for algebraic knots, and extend Galkin-Stohr L-functions. This is directly related to compactified Jacobians of plane curve singularities. Finally, the functional equation and Riemann hypothesis for motivic superpolynomials will be discussed.

(5) *Pavel Etingof* (Massachusetts Institute of Technology, Cambridge, MA, USA)
Title: Analytic Langlands correspondence over $\mathbb{C}$ and $\mathbb{R}$
Abstract: I will review the analytic component of the geometric Langlands correspondence, developed recently in my joint work with *E. Frenkel* and *D. Kazhdan* (based on previous works by other authors), with a special focus on archimedian local fields, especially $\mathbb{R}$. This is based on our work with *E. Frenkel* and *D. Kazhdan* and insights shared by *D. Gaiotto* and *E. Witten*.

(6) *Tamara Grava* (University of Bristol, Bristol, UK and SISSA, Trieste, Italy)
Title: The Stieltjes-Fekete problem and degenerate orthogonal polynomials
Abstract: A result of Stieltjes famously relates the zeroes of the classical orthogonal polynomials with the configurations of points on the line that minimize a suitable logarithmic energy, or equivalently the solutions of a suitable weighted Fekete problem. The optimal configuration satisfies an algebraic set of equations with the logarithmic derivative of the weight function as “external field": we call this set of algebraic equations the Stieltjes–Fekete problem. In this work we consider the Stieltjes-Fekete problem with an arbitrary rational external field. We show that its solutions are in one-to-one correspondence with the zeroes of certain non-hermitean orthogonal polynomials that satisfy an excess of orthogonality conditions and are thus termed “degenerate". This generalizes the above mentioned result of Stieltjes.

(7) *Alexander Its* (Indiana University-Purdue University, Indianapolis, IL, USA)
Title: TT*-equations of Cecotti and Vafa, Riemann-Hilbert method and Iwasawa factorization
Abstract: In this talk, some interesting (we believe !) new features of the Riemann-Hilbert method of the asymptotic analysis of integrable systems will be discussed. These features have emerged during the study of the global solution of the tt* equations of Cecotti and Vafa which the speaker has been pursuing, for some time already, together with *Martin Guest* and *Chang-Shou Lin*. The method which we use in this study is based on a combination of the isomonodrtomy technique and Iwasawa factorization from the theory of loop groups. This allows us to simplify significantly the asymptotic analysis of the tt* equations and, simultaneously, to bring a new light on some aspects of the well known relation between the Birkhoff-Grothendieck and Iwasawa factorizations. Another important link — to a 1980 paper *by Igor Krichever* on the nonlinear analog of the d’Alembert’s formula, will be also highlighted. The talk represents an ongoing joint project of *M. Guest*, *I. Krichever*, and the speaker.

(8) *Alisa Knizel* (University of Chicago, Chicago, IL, USA)
Title: Discrete Painlevé equations and discrete log-gases
Abstract: I will discuss how discrete Painlevé equations appear in the context of discrete log-gases, applications to tiling models and the study of asymptotic properties of these systems.

(9) *Nikita Nekrasov* (Simons Center for Geometry and Physics, Stony Brook University, Stony Brook, NY, USA)
Title: Integrable systems in gauge theories and sigma models
Abstract: I will present a few paintbrushes on the picture of algebraic integrable systems emerging in the two main examples of quantum field theories: two dimensional sigma models, and four dimensional gauge theory. Based on the joint works with Igor Krichever

(10) *Andrei Okounkov* (Columbia University, New York, NY, USA)

Title: Equivariant cobordism and Eisenstein series
Abstract: Eisenstein series produce automorphic forms on a group G from automorphic forms on its Levi subgroups. In an ongoing work with *David Kazhdan*, we use some topological ideas to spectrally decompose the corresponding spaces of automorphic forms, completing certain lines of inquiry that have been around since the pioneering work of Langlands in 1960s. My talk will be an introduction to this subject.

(11) *Duong Phong* (Columbia University, New York, NY, USA)
Title: Geometric flows in symplectic geometry
Abstract: The search for supersymmetric solutions of string theories has led to new geometric partial differential equations which are interesting in their own right. In particular, the Type *IIA* string leads to a new geometric flow, introduced by *T. Fei*, *S. Picard*, *X.W. Zhang*, and the speaker and called the Type *IIA* flow, which is a flow of closed and primitive 3-forms on a symplectic six-dimensional manifold. Remarkably, the Type *IIA* flow can also be viewed as a flow of a special class of almost-complex structures, or as a coupling of the Ricci flow with a scalar field. We survey what is now known about this flow, with emphasis on the many open questions.

(12) *Leon Takhtajan* (Stony Brook University, Stony Brook, NY, USA)
Title: On algebraic de Rham theorem
Abstract: I will discuss algebraic de Rham theorem on curves and its generalization to higher order differentials.

(13) *Alexander Varchenko* (University of North Carolina, Chapel Hill, NC, USA)
Title: Polynomial solutions of KZ equations modulo an integer
Abstract: I will review the construction of polynomial solutions of KZ equations modulo an integer and the properties of the solutions, in particular, their p-adic limit.

(14) *Alexander Veselov* (Loughborough University, Loughborough, UK)
Title: Theta divisors and Chern-Dold character in complex cobordisms
Abstract: The role of the theta functions of Jacobi and Prym varieties in soliton theory is well-known since 1970s. In the recent years this link was used by *Igor Krichever* and collaborators to make a remarkable progress in the corresponding Riemann-Schottky type problems. In the talk I will discuss the case of the general principally polarized abelian varieties and show that the corresponding theta divisors can be chosen as irreducible algebraic representatives of the coefficients of the Chern-Dold character in complex cobordisms. The talk is based on a joint work with *V.M. Buchstaber*.

(15) *Paul Wiegmann* (University of Chicago, Chicago, IL, USA)
Title: Chiral anomalies in hydrodynamics
Abstract: Axial-current anomaly is the fundamental property of quantum field theory with Dirac fermions coupled to a vector gauge field. In quantum electrodynamics (QED), for example, the anomaly states that the divergence of the axial current does not vanish as it would follow from the classical Dirac equation but is controlled by the electromagnetic field

$$\partial \cdot j_A = \tfrac{1}{2} F \cdot {}^\star F$$

Recently and quite remarkably the axial-current anomaly has been identified in the hydrodynamic of a classical ideal fluid as a kinematic property of the Euler equation. In fluid, the axial current is closely related to the fluid helicity and in a charged fluid its divergence (in appropriate units) is identical to that of the axial current of quantum Dirac fermions in QED. The talk is based on the joint works with *A. Abanov*.

(16) *Dmitry Zakharov* (Central Michigan University, Mount Pleasan, MI, USA)

Title: The tropical Prym variety

Abstract: The Prym variety is a principally polarized abelian variety (ppav) associated to an unramified double cover of algebraic curves. In tropical geometry, algebraic curves are replaced with metric graphs, and the Jacobian of a metric graph is a tropical ppav, which is a real torus equipped with an integral structure. I will talk about a tropical version of the Prym construction, which associates a tropical ppav to a double cover of metric graphs. I will prove a volume formula for the tropical Prym variety, which is an analogue of Kirchhoff's matrix tree theorem. This formula can be interpreted in terms of a canonical polyhedral decomposition of the tropical Prym variety (a similar decomposition for the tropical Jacobian was found by Mikhalkin—Zharkov and An—Baker—Kuperberg—Shokrieh). I will also discuss the tropical analogues of the trigonal and tetragonal constructions of Recillas and Donagi. Joint work with *Yoav Len* and *Felix Roehrle*.