Isomonodromic Deformations, Painlevé Equations, and Integrable Systems

Conference Program

(Times slots are in the Eastern Time Zone; UTC-4)

Monday, June 27

8:00–9:00	*Alexander Bobenko*	**The Bonnet problem: Is a surface characterized by its metric and curvatures?**
9:15–10:15	*Marco Bertola*	**The Riemann—Hilbert problem on higher genus surfaces and some applications**
10:30–11:30	*Peter Miller*	**On the algebraic solutions of the Painlevé-III (D_7) equation**

Tuesday, June 28

8:00–9:00	*Yasuhiko Yamada*	**Geometry of quantum discrete Painlevé equations**
9:15–10:15	*Masatoshi Noumi*	**Eigenfunctions for the elliptic Ruijsenaars difference operators**
10:30–11:30	*Alexander Goncharov*	**Noncommutative cluster varieties and noncommutative spaces of Stokes data**
11:45–12:45	*Philippe Di Francesco*	**From Koornwinder operators to cluster algebra: Proof of the Macdonald-Q-system conjecture**

Wednesday, June 29

8:00–9:00	*Alba Grassi*	**Kyiv Formula and String Dualities**
9:15–10:15	*Vladimir Fock*	**Around discrete flows of GK integrable systems**
10:30–11:30	*Nikita Nekrasov*	**Isomonodromic Deformations, Painlevé Equations, and Integrable Systems from $4d$ gauge theory**

Thursday, June 30

8:00–9:00	*Thomas Bothner*	**The complex elliptic Ginibre ensemble at weak non-Hermiticity**
9:15–10:15	*Mattia Cafasso*	**Integrability of an integro-differential Painlevé equation**
10:30–11:30	*Rod Halburd*	**Integrable delay-differential equations**
11:45–12:45	*Leon Takhtajan*	**Symplectic structures on the moduli spaces of curves and bundles**

Friday, July 1

9:15–10:15	*Alexei Borodin*	**Biased 2×2 periodic Aztec diamond and an elliptic curve**
10:30–11:30	*Alisa Knizel*	**Stationary measure for the open KPZ equation**
11:45–12:45	*Axel Saenz*	**A conjecture on the fluctuations for the edge of the Heisenberg-Ising XXZ spin-$1/2$ chain**

Abstracts

(1) *Marco Bertola* (Concordia University, Montreal, Canada)

Title: The Riemann—Hilbert problem on higher genus surfaces and some applications

Abstract: Painlevé transcendents as well as solutions of nonlinear waves are deeply linked to solutions of Riemann—Hilbert problems on the sphere. At their core, these problems define a (trivial) vector bundle on the sphere, and the poles of the transcendents correspond to non-trivial bundles where the partial indices of the associated problem become non-zero. In higher genus there are additional issues linked to the index; the role of degree—zero bundles is better played by degree ng (with n the rank and g the genus). The practical application of the theory of infinitesimal variations then requires a matrix version of the Cauchy kernel that contains as parameters the Turin data, namely the moduli of a reference bundle. While these notions seem closer to algebraic geometry than to Integrable Systems, I will indicate how they become necessary to address certain problems stemming from asymptotic analysis of Padé approximations on Riemann surfaces.

(2) *Alexander Bobenko* (Institut für Mathematik, Technische Universität Berlin, Germany)

Title: The Bonnet problem: Is a surface characterized by its metric and curvatures?

Abstract: We consider a classical problem in differential geometry, known as the Bonnet problem, whether a surface in three space is characterized by its metric and mean curvature function. Generically, the answer is yes. Special cases when it is not the case are classified. The talk consists of two parts. In the first part we consider Bonnet surfaces, which are surfaces (with non-constant mean curvature) possessing continuous families of isometries preserving mean curvature. Their global classification is given using the theory of Painlevé equations. In the second part, which is a joint work with *Tim Hoffmann* and *Andrew Sageman-Furnas*, we explicitly construct a pair of immersed tori that are related by a mean curvature preserving isometry. This resolves a longstanding open problem on whether the metric and mean curvature function determine a unique compact surface. Discrete differential geometry is used to find crucial geometric properties of surfaces.

(3) *Alexei Borodin* (Massachusetts Institute of Technology, USA)

Title: Biased 2×2 periodic Aztec diamond and an elliptic curve

Abstract: We study a biased 2×2 periodic random domino tilings of the Aztec diamond and associate a linear flow on an elliptic curve to this model. Our main result is a double integral formula for the correlation kernel, in which the integrand is expressed in terms of this flow. For special choices of parameters the flow is periodic, and this allows us to perform a saddle point analysis for the correlation kernel. In these cases we compute the local correlations in the smooth disordered (or gaseous) region. The special example in which the flow has period six is worked out in more detail, and we show that in that case the boundary of the rough disordered region is an algebraic curve of degree eight. Joint work with *Maurice Duits*.

(4) *Thomas Bothner* (King's College London, London, UK)

Title: The complex elliptic Ginibre ensemble at weak non-Hermiticity

Abstract: In this talk we will focus on the complex elliptic Ginibre ensemble (*eGinUE*) and analyze the statistical behavior of its eigenvalues in a suitable scaling limit, known as the weak non-Hermiticity limit. In this limit the asymmetry parameter in the model scales with the matrix dimension and the so obtained $2D$ limiting point processes generalize the well-known sine and Airy processes from the Gaussian unitary ensemble. Using integro-differential Painlevé transcendents we will show how the gap functions of the $2D$ limiting point processes can be evaluated in closed form and how Riemann-Hilbert techniques can subsequently yield precise asymptotic information for the same functions. Based on current joint work with *Alex Little*.

(5) *Mattia Cafasso* (Université d'Angers, Angers, France)

Title: Integrability of an integro-differential Painlevé equation

Abstract: During my talk, I will discuss some of the integrability properties of an integro-differential version of the Painlevé-II equation, recently appeared in the theory of integrable probability, KPZ equation and non-interacting fermions. The results I will discuss have been obtained in collaboration with *Thomas Bothner*, *Tom Claeys*, *Giulio Ruzza*, and *Sofia Tarricone*.

(6) *Philippe Di Francesco* (University of Illinois at Urbana-Champaign, Urbana, IL, USA)

Title: From Koornwinder operators to cluster algebra: Proof of the Macdonald-Q-system conjecture

Abstract: We present various constructions of commuting difference operators for the theory of Koornwinder polynomials. We show how a specialization/limiting procedure produces a functional representation for quantum Q-system cluster algebras associated to affine and twisted types A, B, C, D, also interpreted as discrete algebraic quantum integrable systems. The correspondence uses Koornwinder duality and a suitable Fourier-Whittaker transform allowing to interpret Koornwinder polynomial Pieri rules as relativistic Toda systems. (Based on joint work with *Rinat Kedem*).

(7) *Vladimir Fock* (Université de Strasbourg et CNRS, Strasbourg, France)

Title: Around discrete flows of GK integrable systems

Abstract: The flows of Goncharov-Kenyon integrable systems contains a discrete subflow for which the evolution is rational. The best known example of this flow is the Poncelet porism. We will describe this flow from different points of view, give explicit expression for them and in particular we will show that Poncelet construction can be generalized to any GK system.

(8) *Alexander Goncharov* (Yale University, New Haven CT, USA)

Title: Noncommutative cluster varieties and non-commutative spaces of Stokes data

Abstract: I will explain what are non-commutative cluster varieties using configurations of triples of flags/decorated flags in an n-dimensional vector space over a non-commutative field, aka skew field. Moduli spaces of non-commutative Stokes data are examples of non-commutative cluster varieties. The talk is based on a joint work with *Maxim Kontsevich*.

(9) *Alba Grassi* (UniversitÉ de Genève and CERN, Geneva, Switzerland)

Title: Kyiv Formula and String Dualities

Abstract: During the last decades string theory and its dualities have led to new insights and beautiful results in mathematical physics. In this talk I will focus on one particular example of such duality: the one relating topological string theory and spectral theory. I will show that, by using Kyiv formula, we can prove some aspects of this correspondence. In addition I will show how the interaction between these topics leads to new results in the context of q-Painlevé equations.

(10) *Rod Halburd* (University College London, London, UK)

Title: Integrable delay-differential equations

Abstract: We will study several types of delay-differential equations (equations for a function of a single variable z involving shifts and derivatives with respect to z). We find equations admitting multi-parameter families of elliptic function solutions. We will also study a family of delay-differential equations and show that the demand that there is a non-trivial finite-order meromorphic solution with sufficiently many zeros eliminates all but a small number of equations, some of which have continuum limits to differential Painlevé equations and are the compatibility conditions of related linear problems. Joint work with *Bjorn Berntson* and *Risto Korhonen*.

(11) *Alisa Knizel* (University of Chicago, Chicago, IL, USA)

Title: Stationary measure for the open KPZ equation

Abstract: The Kardar-Parisi-Zhang (*KPZ*) equation is the stochastic partial differential equation that models interface growth. In the talk I will present the construction of a stationary measure for the KPZ equation on a bounded interval with general inhomogeneous Neumann boundary conditions. Along the way, we will encounter classical orthogonal polynomials, the asymmetric simple exclusion process, and precise asymptotics of q-Gamma functions. This construction is a joint work with *Ivan Corwin*.

(12) *Peter Miller* (The University of Michigan, Ann Arbor, MI, USA)

Title: On the algebraic solutions of the Painlevé-III (D_7) equation

Abstract: The D_7 degeneration of the Painlevé-III equation has solutions that are rational functions of $x^{1/3}$ for certain parameter values. We apply the isomonodromy method to obtain a Riemann-Hilbert representation of these solutions. We demonstrate the utility of this representation by analyzing rigorously the behavior of the solutions in the large parameter limit. This is joint work with *Robert Buckingham* (University of Cincinnati).

(13) *Nikita Nekrasov* (Simons Center for Geometry and Physics, Stony Brook University, Stony Brook, NY, USA)

Title: Isomonodromic Deformations, Painlevé Equations, and Integrable Systems from $4d$ gauge theory

Abstract: I will give a short overview of the field.

(14) *Masatoshi Noumi* (Rikkyo University, Tokyo, Japan)

Title: Eigenfunctions for the elliptic Ruijsenaars difference operators

Abstract: On the basis of a collaboration with *Edwin Langmann* (KTH) and *Junichi Shiraishi* (Tokyo), I report recent progresses in understanding the joint eigenfunctions for the commuting family of elliptic Ruijsenaars difference operators. After reviewing some basic known facts regarding the Macdonald-Ruijsenaars operators in the trigonometric case, I propose two classes of joint eigenfunctions for the elliptic Ruijsenaars operators:
(A) symmetric eigenfunctions around the torus that deform Macdonald polynomials, and
(B) asymptotically free eigenfunctions in a certain asymptotic domain.

(15) *Axel Saenz* (Oregon State University, Corvalis, OR, USA)

Title: A conjecture on the fluctuations for the edge of the Heisenberg-Ising XXZ spin-$1/2$ chain

Abstract: In a recent collaboration with *Craig Tracy* and *Harold Widom*, we presented a conjecture for the distribution for the edge of the XXZ spin-$1/2$ chain with domain wall initial conditions. The conjecture is based on Bethe Ansatz contour integral formulas and the method of steepest descent. In the

end, we arrive at Fredholm-series-like limit that we were unable to compute. We conjecture that this limit exist and thus gives the distribution of the edge of the *XXZ* spin-$1/2$ chain with domain wall initial conditions. In the talk, I will give further details of this conjecture and show certain computations related to the conjecture.

(16) *Leon Takhtajan* (Stony Brook University, Stony Brook, NY, USA)

Title: Symplectic structures on the moduli spaces of curves and bundles

Abstract: I will review old and new results on symplectic properties of the moduli spaces of curves and bundles, accentuating their similarities and differences.

(17) *Yasuhiko Yamada* (Kobe University, Kobe, Japan)

Title: Geometry of quantum discrete Painlevé equations

Abstract: We study a representation of some affine Weyl groups given by birational actions on quantum variables x, y such that $yx = qxy$. The polynomials $F(x, y)$ arising from the Weyl group actions are studied and their geometric characterization is given. Based on the geometric picture, we give a bilinear form of the Weyl group representation. An application to the quantum mirror curves will be also discussed. In this talk, we will mainly consider the case of type $E_8^{(1)}$.