PATH – DEPENDENT OPTIONS ON ASSETS

WITH JUMPS AND A NEW APPROACH

TO CREDIT DEFAULT SPREADS

BY

ALEXANDER LIPTON

FX PRODUCT DEVELOPMENT

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1. BACKGROUND

Deviations from the Black-Scholes paradigm are ubiquitous.

They result in volatility smiles and smirks for all asset classes.

Usually smiles and smirks are explained by stochastic or static volatilities or their combinations.

However, jumps are very important but very difficult to handle properly.

Early paper by Merton (1976): European options on assets driven by jump-diffusions with log-normal Poissonian jumps.

Merton's model is complemented by Kou (2000): European options on assets driven by jump-diffusions with log-double-exponential Poissonian jumps.

More general approach, for instance, Madan and Seneta (1990) and many others: European options on assets driven by Levy processes.

Andersen and Andreassen (2000) priced European options on jump-diffusions numerically.

Papers on EXOTIC options are rather limited.

However, several important papers should be mentioned:

Mordecki (1999) (perpetual American); Nahum (1999) (lookbacks); Boyarchenko and Levendorsky (2000), Kou and Wang (2001), Avram *et al.* (2001) (barriers and lookbacks); Boyarchenko (2001) (credit default swaps).

Existing formulas are VERY complicated even for numerical evaluation and require further refinement.

We develop a NEW approach to pricing of path-dependent options on assets driven by jump-diffusions with log-exponential Poissonian jumps.

Our approach is based on celebrated FLUCTUATION IDENTITIES for Levy processes.

Conceptually it is rather complex but computationally it is simple.

As a by-product, we obtain a new formula for CREDIT DEFAULT SPREADS which complements (and improves) a recent formula given by Finkelstein *et al*. (2001).

Details are given in Lipton (2001 a,b)

2. FORMULATION

Consider a generic underlying asset driven by the SDE

$$\frac{dS(t)}{S(t)} = \left(r^{01} - \nu\eta\right) dt + \sigma dW(t) + \left(e^{J} - 1\right) dN(t),$$

W is a standard Wiener process, N is a Poisson process with intensity ν, and J is the jump magnitude which is assumed to be a random variable with the probability density function (p.d.f.) of the form $\varpi(J)$, $r^{01} = r^{0} - r^{1}$, r^{0}, r^{1} are the risk-neutral and dividend rates, respectively, and η is the average magnitude of the jump,

$$\eta = \mathbb{E}\left\{e^{J} - 1\right\} = \int_{-\infty}^{\infty} e^{J}\varpi(J)\, dJ - 1.$$

The log-process $\xi(t) = \ln\left(S(t)/S_0\right)$ is driven by the SDE

$$d\xi(t) = \mu dt + \sigma dW(t) + J dN(t), \qquad \xi(0) = 0,$$

where $\mu = r^{01} - \nu\eta - \sigma^2/2$.

It is easy to price a European option on such a process but VERY difficult to price a path-dependent options.

The reason is simple: the distribution of the value of the process is readily available but the distribution of its maximum is not.

Probabilistic functions which we need to know are:

The cumulative p.d.f. (c.p.d.f.) for the value of the process at time t:

$$F(t,x) = \mathbb{P}\{\xi(t) < x\},$$

The corresponding p.d.f. $f(t,x) = dF(t,x)/dx$

The c.p.d.f.'s $Q_{\pm}(t,x)$ for the maximum and minimum of the process,

$$Q_{+}(t,x) = \mathbb{P}\left\{\sup_{0 \le s \le t} \xi(s) < x\right\}, \qquad x > 0,$$

$$Q_{-}(t,x) = \mathbb{P}\left\{\inf_{0\le s\le t} \xi(s) < x\right\}, \qquad x < 0,$$

The corresponding p.d.f.'s $q_{\pm}(t,x) = dQ_{+}(t,x)/dx$

Auxiliary p.d.f.'s for the first passage time at a level x, τ_x, and the size of the overshot, γ_x,

$$R(t,x,y) = \mathbb{P}\{\tau_x \in dt, \gamma_x \in (y,\infty)\} \qquad y \ge 0,$$

$$\begin{aligned} r(t,x) &= \mathbb{P}\{\tau_x \in dt, \gamma_x = 0\} \\ &= \frac{d}{dt}(1 - Q_{+}(t,x)) - R(t,x,0). \end{aligned}$$

The joint c.p.d.f.

$$H(t,a,x) = \mathbb{P}\left\{\sup_{0\le s\le t} \xi(s) < a, \xi(t) < x\right\}, a > 0, x \le a.$$

Its density $h(t,a,x) = dH(t,a,x)/dx$.

For brevity, we concentrate on *crossing positive barriers from below* and assume that the process $\xi(t)$ can have only *positive* exponentially distributed jumps, i.e.,

$$\varpi(J) = \begin{cases} \varphi e^{-\varphi J}, & J \geq 0, \\ 0, & J < 0, \end{cases}$$

$\varphi > 1$, and show how to evaluate $Q_{\pm}(t,x)$, and $H(t,a,x)$ explicitly.

The expectation of J and its volatility are equal to $1/\varphi$, while $\eta = 1/(\varphi - 1)$.

It is relatively easy to study the hyperexponential case.

3. EUROPEAN OPTIONSON LEVY PROCESSES

The Fokker-Planck equation for the transitional p.d.f. (t.p.d.f.) $f(t,x)$:

$$f_t - \frac{1}{2}\sigma^2 f_{xx} + \mu f_x - \nu \int_{-\infty}^{\infty} [f(x-J) - f(x)] \varpi(J) \, dJ = 0,$$

$$f(0,x) = \delta(x).$$

Introduce its characteristic function $\tilde{f}$,

$$f(t,x) \to \tilde{f}(t,z) = \int_{-\infty}^{\infty} e^{ixz} f(t,x) \, dx.$$

Obtain an ordinary differential equation (ODE) for $\tilde{f}$:

$$\tilde{f}_t - K(z) \tilde{f} = 0, \qquad \tilde{f}(0,z) = 1.$$

Here $K(z)$ is the so-called cumulant of the process $\xi(t)$,

$$K(z) = -\frac{1}{2}\sigma^2 z^2 + \mu i z + \nu \int_{-\infty}^{\infty} \left(e^{iJz} - 1\right) \varpi(J) \, dJ.$$

Using exponential $\varpi(J)$ yields

$$K(z) = -\frac{1}{2}\sigma^2 z^2 + \mu i z + \nu \frac{iz}{\varphi - iz}.$$

Accordingly,

$$\tilde{f}(t, z) = \int_{-\infty}^{\infty} e^{ixz} f(t, x)\, dx = e^{tK(z)},$$

$$f(t, x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ixz + tK(z)} dz.$$

This a slightly simplified version of the Levy-Khintchine formula which effectively solves the governing SDE.

These formulas can be used for pricing European options.

Consider an option with maturity T and a generic payoff $V(S)$.

Introduce $v(x) = V(S_0 e^x)/S_0$ and represent the price at inception as

$$\begin{aligned} V_0 &= e^{-r^0 T} S_0 \int_{-\infty}^{\infty} f(T,x)\, v(x)\, dx \\ &= \frac{e^{-r^0 T} S_0}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-ixz+TK(z)} v(x)\, dx dz \\ &= \frac{e^{-r^0 T} S_0}{2\pi} \int_{-\infty}^{\infty} e^{TK(z)} \overline{\tilde{v}(z)} dz. \end{aligned}$$

Here $\tilde{v}(z)$ is the Fourier transform of $v(x)$, and $\overline{\tilde{v}(z)}$ is its complex conjugate.

It is easy to compute $\tilde{v}(z)$ for calls and puts and all other standard European options.

This formula solves the pricing problem for European options.

It is easy to apply this formula to jump-diffusions with stochastic volatility.

For a European call struck at K this formula assumes the form

$$\begin{aligned} C_0 &= e^{-r^1 T} S_0 - \frac{e^{-r^0 T} K}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-iz\kappa + T\mathcal{K}(z)}}{-iz+1} dz \\ &\quad - \frac{e^{-r^0 T} K}{2\pi} \text{P.V.} \int_{-\infty}^{\infty} \frac{e^{-iz\kappa + T\mathcal{K}(z)}}{iz} dz, \end{aligned}$$

$\kappa = \ln\left(K/S_0\right).$

Here P.V. stands for the principal value of the integral.

Equivalent formulas are given by Lipton (2000) and Lewis (2001), among others.

In the limit $\nu \to 0$ this formula reduces to the familiar Black-Scholes form.

For benchmarking purposes it is useful to evaluate the distribution function $f\left(t, x\right)$ directly.

We can represent $f(t,x)$ as follows

$$f(t,x) = \sum_{n=0}^{\infty} \frac{e^{-\nu t} (\nu t)^n}{n!} f_n(t,x).$$

Here $f_n(t,x)$ are conditional p.d.f. corresponding to the arrival of exactly n jumps between 0 and t.

The characteristic functions of $f_n(t,x)$ have the form

$$\tilde{f}_n(t,z) = \frac{(i\varphi)^n e^{i\mu t z - \sigma^2 t z^2/2}}{(z+i\varphi)^n},$$

$$f_n(t,x) = \frac{(i\varphi)^n}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-i(x-\mu t)z - \sigma^2 t z^2/2}}{(z+i\varphi)^n} dz.$$

Integration by parts yields

$$\begin{aligned} f_n(t,x) &= \frac{\varphi(x-\mu t)-\varphi^2\sigma^2 t}{(n-1)} f_{n-1}(t,x) \\ &\quad + \frac{\varphi^2\sigma^2 t}{(n-1)} f_{n-2}(t,x). \end{aligned}$$

We can evaluate $f_n(t,x)$ recursively provided that we know $f_0(t,x)$, $f_1(t,x)$.

$$f_0(t,x) = \frac{e^{-(x-\mu t)^2/2\sigma^2 t}}{\left(2\pi\sigma^2 t\right)^{1/2}},$$

$$\begin{aligned} f_1(t,x) &= \varphi e^{-\varphi(x-\mu t)+\varphi^2\sigma^2 t/2} \\ &\quad \times N\left(\frac{x-\mu t}{\left(\sigma^2 t\right)^{1/2}} - \varphi\left(\sigma^2 t\right)^{1/2}\right). \end{aligned}$$

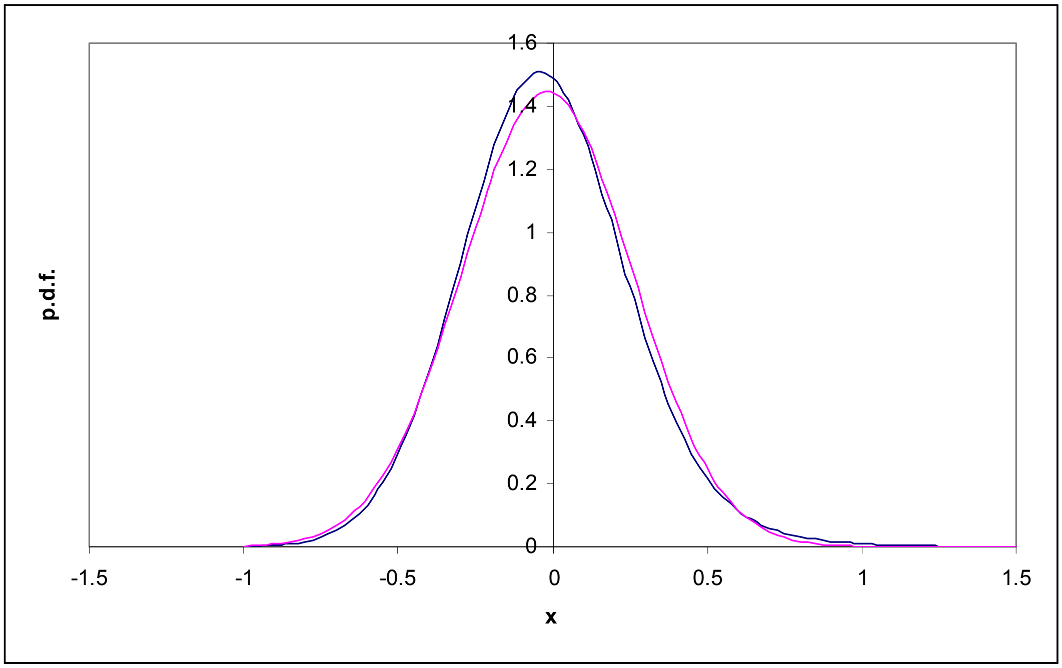

Figure 1: A typical p.d.f. for a jump diffusion with log-exponential jumps. For comparison, the p.d.f. for a regular diffusion with equivalent parameters is also shown. Fat tails and sharp peaks are clearly seen.

4. THE FLUCTUATION IDENTITIES

It is difficult to deal with the relevant distributions directly but somewhat easier to deal with their Laplace (or Laplace-Carson) transforms.

We introduce

$$\begin{aligned} \mathfrak{F}(\lambda, x) &= \int_0^\infty e^{-\lambda t} F(t, x)\, dt, \\ \mathfrak{f}(\lambda, x) &= \int_0^\infty e^{-\lambda t} f(t, x)\, dt, \end{aligned}$$

$$\begin{aligned} \mathfrak{Q}_\pm(\lambda, x) &= \int_0^\infty Q_\pm(t, x)\, e^{-\lambda t} \lambda dt, \\ \mathfrak{q}_\pm(\lambda, x) &= \int_0^\infty q_\pm(t, x)\, e^{-\lambda t} \lambda dt, \end{aligned}$$

$$\mathfrak{R}(\lambda, a, x) = \int_0^\infty e^{-\lambda t} R(t, a, x)\, dt,$$

$$\begin{aligned} \mathfrak{r}(\lambda, a) &= \int_0^\infty e^{-\lambda t} r(t, a)\, dt \\ &= 1 - \mathfrak{Q}_+(\lambda, a) - \mathfrak{R}(\lambda, a, 0), \end{aligned}$$

$$\mathfrak{H}(\lambda, a, x) = \int_0^\infty e^{-\lambda t} H(t, a, x)\, dt,$$
$$\mathfrak{h}(\lambda, a, x) = \int_0^\infty e^{-\lambda t} h(t, a, x)\, dt,$$

$$\widetilde{\mathfrak{q}}_\pm(\lambda, z) = \int_{-\infty}^\infty e^{ixz} \mathfrak{q}_\pm(\lambda, x)\, dx.$$

The key to what follows are the celebrated fluctuation identities:

$$\widetilde{\mathfrak{q}}_+(\lambda, z) = \exp\left\{\int_0^\infty \frac{e^{-\lambda t}}{t}\left(\int_0^\infty \left(e^{ixz} - 1\right) f(t, x)\, dx\right) dt\right\},$$

$$\widetilde{\mathfrak{q}}_-(\lambda, z) = \exp\left\{\int_0^\infty \frac{e^{-\lambda t}}{t}\left(\int_{-\infty}^0 \left(e^{ixz} - 1\right) f(t, x)\, dx\right) dt\right\}.$$

The first passage time τ_a has the form

$$\mathbb{E}\left\{e^{-\lambda \tau_a}\right\} = 1 - \mathfrak{Q}_+(\lambda, a).$$

Provided that both $\mathfrak{q}_{+}(\lambda, a)$, $a > 0$, and $\mathfrak{q}_{-}(\lambda, b)$, $b < 0$, are known, we represent $\mathfrak{R}(\lambda, a, y)$ as

$$\begin{aligned}\mathfrak{R}(\lambda, a, y) &= \frac{1}{\lambda}\int_{0}^{a}\int_{-\infty}^{0} M(a+y-u-v) \\ &\quad \times \mathfrak{q}_{-}(\lambda, u)\,\mathfrak{q}_{+}(\lambda, v)\,du dv.\end{aligned}$$

Here

$$M(x) = \nu\int_{x}^{\infty}\varpi(y)\,dy, \qquad x > 0,$$

A simple “bean counting” suggests that for $x < a$ that

$$\begin{aligned}\mathfrak{H}(\lambda, a, x) &= \mathfrak{F}(\lambda, x) + \int_{0}^{\infty}\mathfrak{F}(\lambda, x-a-y)\,d\mathfrak{R}(\lambda, a, y) \\ &\quad -\mathfrak{F}(\lambda, x-a)\left(1 - \mathfrak{Q}_{+}(\lambda, a) - \mathfrak{R}(\lambda, a, 0)\right).\end{aligned}$$

$$\begin{aligned}\mathfrak{h}(\lambda, a, x) &= \mathfrak{f}(\lambda, x) + \int_{0}^{\infty}\mathfrak{f}(\lambda, x-a-y)\,d\mathfrak{R}(\lambda, a, y) \\ &\quad -\mathfrak{f}(\lambda, x-a)\left(1 - \mathfrak{Q}_{+}(\lambda, a) - \mathfrak{R}(\lambda, a, 0)\right).\end{aligned}$$

Once $\mathfrak{H}(\lambda, a, x)$ and $\mathfrak{h}(\lambda, a, x)$ are determined, we can find $H(t, a, x)$ and $h(t, a, x)$ via the inverse Laplace transform.

Three methods for inversion of the Laplace transform:

1. Bromwich inversion integral

$$f(t) = \frac{1}{2\pi i} \int_{b-i\infty}^{b+i\infty} e^{\lambda t} \mathfrak{f}(\lambda)\, d\lambda.$$

2. Post-Widder inversion formula

$$f(t) = \lim_{n\to\infty} \frac{(-1)^n}{n!} \left(\frac{n+1}{t}\right)^{n+1} \mathfrak{f}^{(n)}\left(\frac{n+1}{t}\right).$$

3. Laguerre-series representation

$$f(t) = e^{-t/2} \sum_{n=0}^{\infty} q_n L_n(t),$$

$$L_n(t) = \sum_{k=0}^{n} \left(\begin{array}{c} n \\ k \end{array} \right) \frac{(-t)^k}{k!},$$

$$q(z) = \sum_{n=0}^{\infty} q_n z^n = \frac{1}{1-z} \mathfrak{f}\left(\frac{1+z}{2(1-z)}\right).$$

We use Gaver-Stehfest algorithm based on Post-Widder formula.

5. EXPONENTIAL JUMPS

We introduce the function

$$L(\beta) = K(i\beta) = \frac{1}{2}\sigma^2\beta^2 - \mu\beta - \frac{\nu\beta}{\beta + \varphi}.$$

This function is meromorphic in the complex plane with a simple pole at $\beta = -\varphi$.

We consider the characteristic equation

$$L(\beta) = \lambda, \qquad \lambda \geq 0.$$

This equation has three real roots,

$$\beta_1(\lambda) < -\varphi < \beta_2(\lambda) < 0 < \beta_3(\lambda).$$

These roots are shown in Figure 2.

For the standard Brownian motion we have

$L(\beta) = \sigma^2\beta^2/2, \beta_1 = -\left(2\lambda/\sigma^2\right)^{1/2}, \beta_3 = \left(2\lambda/\sigma^2\right)^{1/2}.$

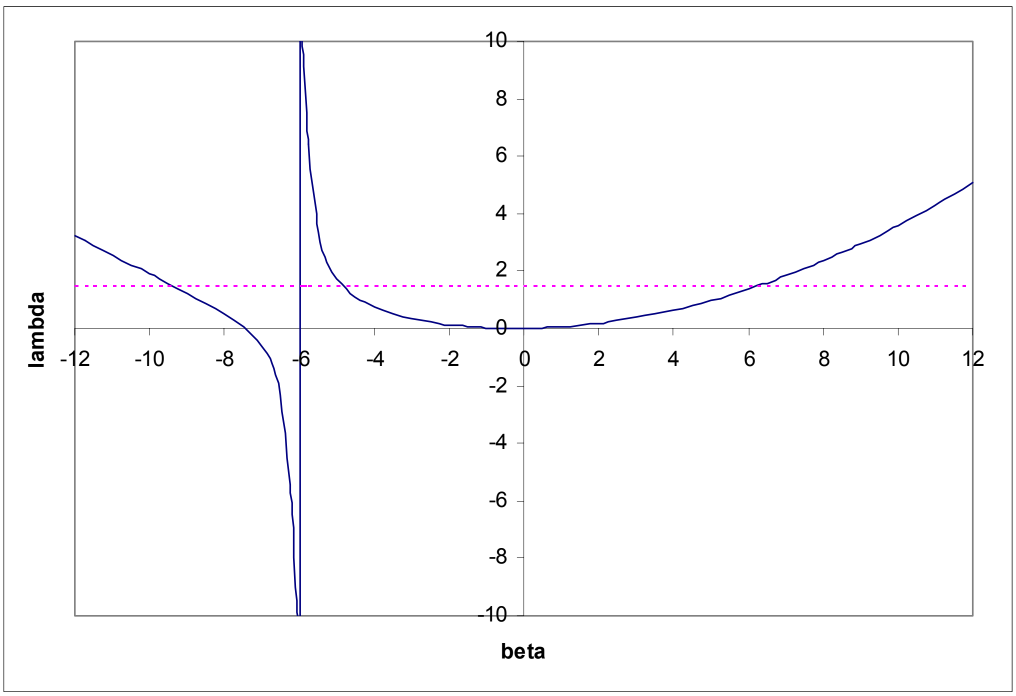

Figure 2: The graphical solution of the characteristic equation.

These roots can be found explicitly via the celebrated Cardano formula:

$$\begin{aligned}
\beta_1 &= 2p^{1/2}\cos\frac{\zeta+2\pi}{3}-\frac{b}{3},\\
\beta_2 &= 2p^{1/2}\cos\frac{\zeta+4\pi}{3}-\frac{b}{3},\\
\beta_3 &= 2p^{1/2}\cos\frac{\zeta}{3}-\frac{b}{3},
\end{aligned}$$

where

$$\begin{aligned}
b &= \varphi-\frac{2\mu}{\sigma^2},\\
c &= -\frac{2\left(\mu\varphi+\nu+\lambda\right)}{\sigma^2},\\
d &= -\frac{2\lambda\varphi}{\sigma^2},\\
p &= \frac{b^2}{9}-\frac{c}{3},\\
q &= \frac{b^3}{27}-\frac{cb}{6}+\frac{d}{2},
\end{aligned}$$

$$\zeta=\left(1+\mathrm{sign}\left(q\right)\right)\frac{\pi}{2}-\mathrm{sign}\left(q\right)\mathrm{atan}\left(p^3/q^2-1\right)^{1/2}.$$

Now we are ready to work with fluctuation identities.

$$\widetilde{\mathfrak{q}}_{+}(\lambda, z) = \frac{i\beta_1\beta_2(-z+i\varphi)}{\varphi(-z+i\beta_1)(-z+i\beta_2)},$$

$$\mathfrak{Q}_{+}(\lambda, a) = 1 - \frac{(\beta_1+\varphi)\beta_2}{\varphi(\beta_2-\beta_1)}e^{\beta_1 a} - \frac{(\beta_2+\varphi)\beta_1}{\varphi(\beta_1-\beta_2)}e^{\beta_2 a},$$

$$\mathbb{E}\left\{e^{-\lambda\tau_a}\right\} = \frac{(\beta_1+\varphi)\beta_2}{\varphi(\beta_2-\beta_1)}e^{\beta_1 a} + \frac{(\beta_2+\varphi)\beta_1}{\varphi(\beta_1-\beta_2)}e^{\beta_2 a}.$$

$$\mathfrak{Q}_{-}(\lambda, b) = e^{\beta_3 b}, \qquad b < 0.$$

The cumulative jump function $M(x)$ is

$$M(x) = \int_x^\infty \nu\varphi e^{-\varphi y}dy = \nu e^{-\varphi x}, \qquad x > 0,$$

Thus,

$$\mathfrak{R}(\lambda, a, y) = \frac{(\beta_1+\varphi)(\beta_2+\varphi)\left(e^{\beta_1 a} - e^{\beta_2 a}\right)e^{-\varphi y}}{\varphi(\beta_2-\beta_1)},$$

$$\mathfrak{r}(\lambda, a) = -\frac{(\beta_1+\varphi)e^{\beta_1 a}}{\beta_2-\beta_1} - \frac{(\beta_2+\varphi)e^{\beta_2 a}}{\beta_1-\beta_2}.$$

A simple calculation yields

$$\mathfrak{f}(\lambda, x) = \begin{cases} -e^{\beta_1 x}/L'(\beta_1) - e^{\beta_2 x}/L'(\beta_2), & x \geq 0, \\ e^{\beta_3 x}/L'(\beta_3) & x < 0. \end{cases}$$

Thus, for $0 \leq x < a$,

$$\begin{aligned} \mathfrak{h}(\lambda, a, x) &= -\frac{e^{\beta_1 x}}{L'(\beta_1)} - \frac{e^{\beta_2 x}}{L'(\beta_2)} \\ &-\frac{2}{\sigma^2 \beta_1}\left(\frac{\beta_1 + \varphi}{\beta_2 - \beta_1} - \frac{\lambda \varphi}{\beta_1 \beta_2 L'(\beta_1)}\right) e^{\beta_1 a + \beta_3 (x-a)} \\ &-\frac{2}{\sigma^2 \beta_2}\left(\frac{\beta_2 + \varphi}{\beta_1 - \beta_2} - \frac{\lambda \varphi}{\beta_1 \beta_2 L'(\beta_2)}\right) e^{\beta_2 a + \beta_3 (x-a)}, \end{aligned}$$

and similar formula for $x < 0$.

A similar formula for $\mathfrak{Q}_+(\lambda, a)$ is given by Kou and Wand (2001) but their formula for $\mathfrak{h}(\lambda, a, x)$ (which is crucial for our approach) is MUCH more complex. We show that both these functions are log-affine.

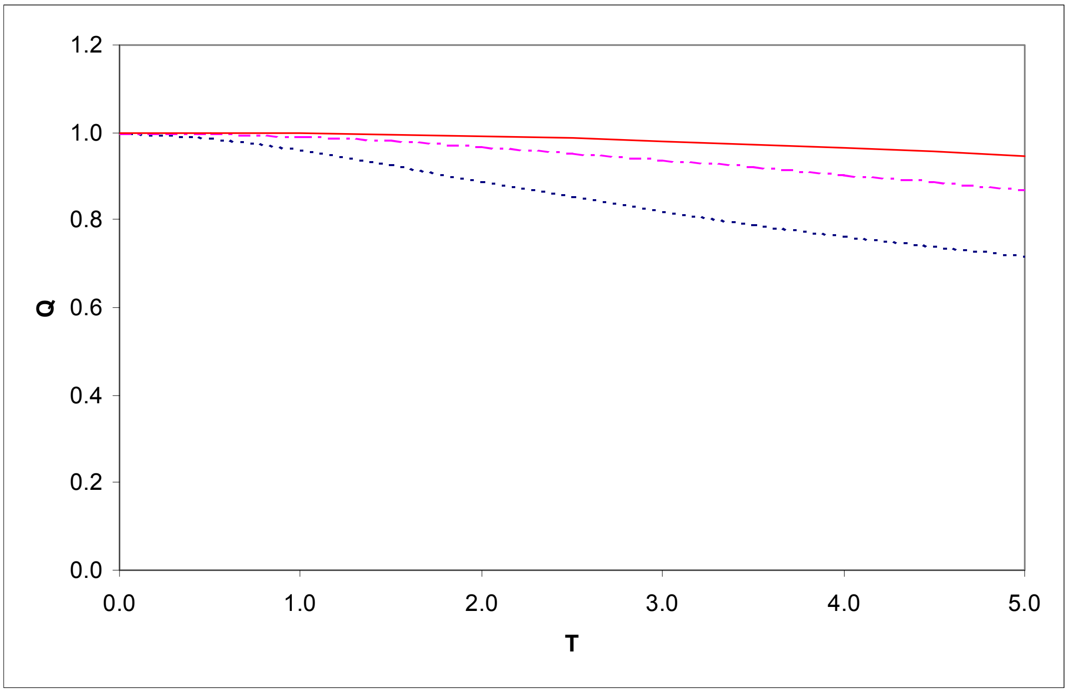

Figure 3: The survival probabilities $Q(t,a)$ for three representative choices of $a = 1.00, 0.75, 0.50$.

6. PRICING OF PATH – DEPENDENT OPTIONS

Since the relevant p.d.f.'s are log-affine finding the Laplace transform of option prices is simple.

Consider for example a lookback option. Its payoff is

$$v(a,x) = e^{a} - e^{x}.$$

The Laplace-transformed undiscounted normalized price of a lookback put $\mathfrak{LBP}(\lambda)$ is

$$\mathfrak{LBP}(\lambda) = \int_0^{\infty}\int_0^{a}\left(e^{a}-e^{x}\right)\frac{d}{da}\mathfrak{h}(\lambda,a,x)\,da\,dx$$
$$= \frac{\beta_2(\beta_1-\beta_3)}{(\beta_1+1)(\beta_3+1)} \times\left(\frac{(\beta_1+\varphi)}{\lambda\varphi(\beta_2-\beta_1)}-\frac{1}{\beta_1\beta_2 L'(\beta_1)}\right)$$
$$+\frac{\beta_1(\beta_2-\beta_3)}{(\beta_2+1)(\beta_3+1)} \times\left(\frac{(\beta_2+\varphi)}{\lambda\varphi(\beta_1-\beta_2)}-\frac{1}{\beta_1\beta_2 L'(\beta_2)}\right).$$

A straightforward application of the Gaver-Stehfest algorithm, discounting and scaling yields the value of a lookback put at inception LBP_0 as a function of its maturity,

$$LBP_0 = e^{-r^0 T} S_0 \mathcal{L}^{(-1)} \{\mathfrak{LBP}(\lambda)\}.$$

A similar formula is valid for barrier call options including reverse knock-outs.

We note that the Gaver-Stehfest algorithm completely solves the pricing problem for Asian, Passport, timer and MANY other options.

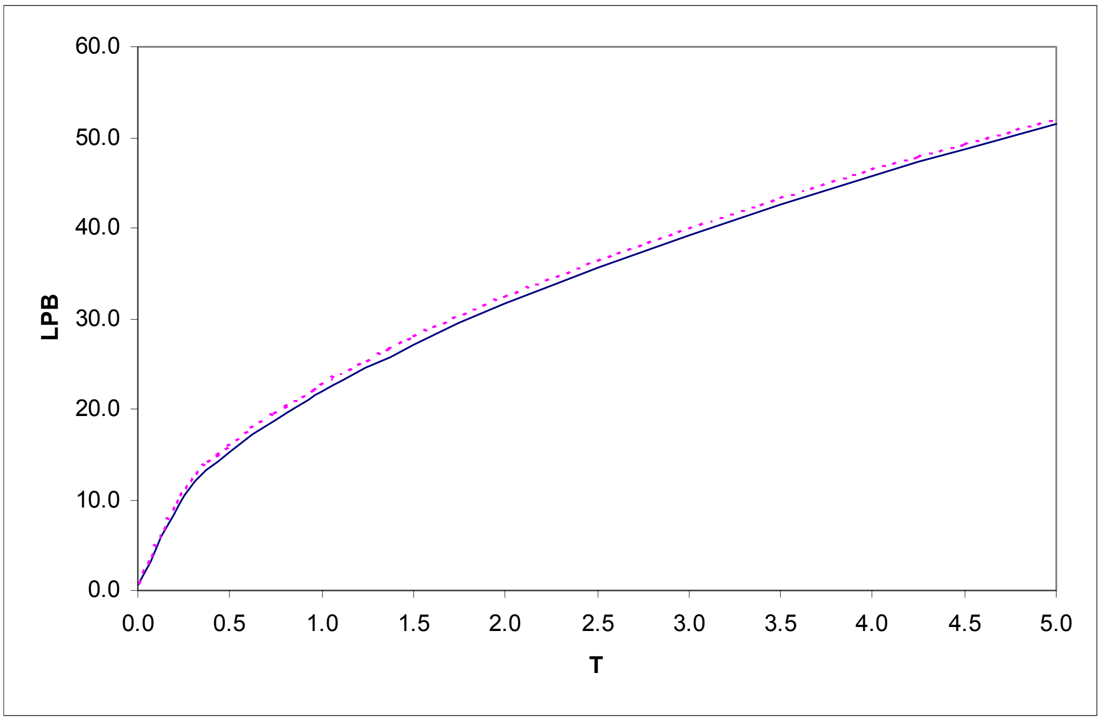

Figure 4: Lookback prices for jump-diffusions and equivalent regular diffusions.

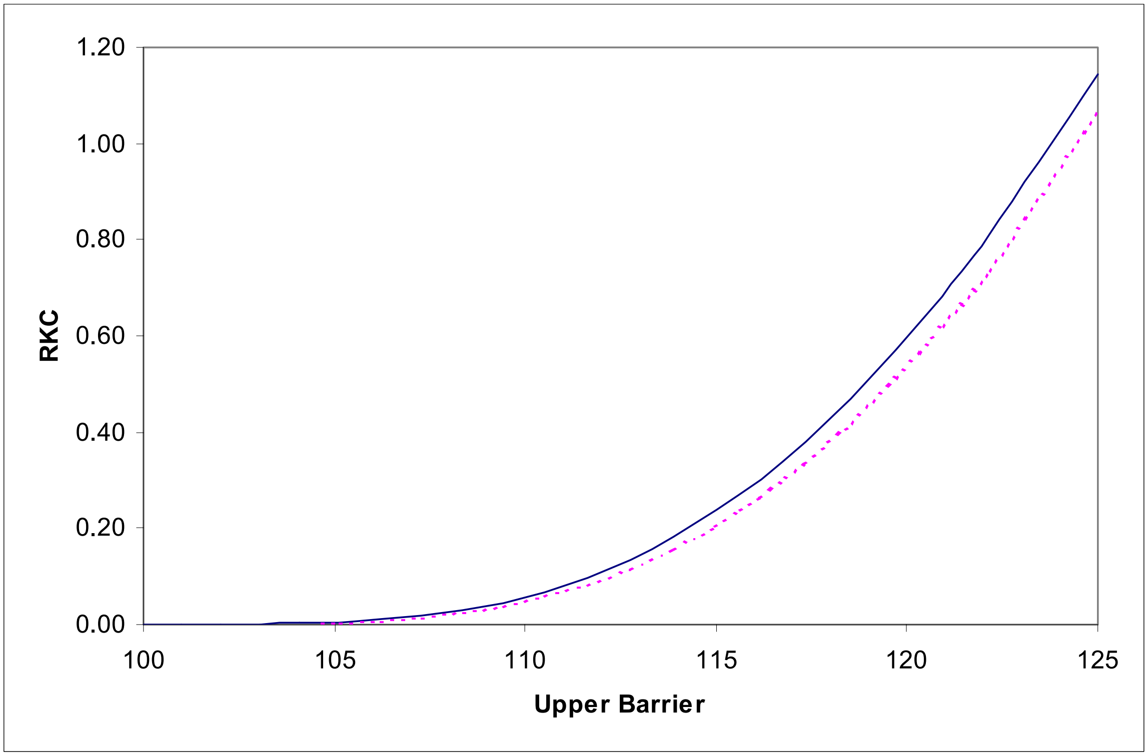

Figure 5: Reverse knock-out prices for jump-diffusions and equivalent regular diffusions.

7. CREDIT DEFAULT SPREADS

We build our theory in the Merton-Leland framework for an individual name whose stock can jump downward.

The value of the firm is $V = S + LD$, where S, D, and L are its stock price, debt-per-share, and global recovery rate of its liabilities, respectively.

In particular, $V_0 = S_0 + LD_0$.

We assume that $V(t)$ is stochastic and is governed by the SDE

$$\frac{dV(t)}{V(t)} = \left(r^{01} - \nu\bar{\eta}\right) dt + \bar{\sigma} dW(t) + \left(e^{-\bar{J}} - 1\right) dN(t),$$

The debt-per-share is deterministic and has the form $D(t) = e^{r^{01}t} D_0$.

The firm defaults if its value hits the barrier $LD(t)$.

Here $\bar{\sigma}$ is the volatility of the firm's value, $\bar{J}$ is its jump size which has an exponential distribution with parameter $\bar{\varphi}$, and $\bar{\eta} = -1/(\bar{\varphi}+1)$.

The process $\chi(t) = \ln(V_0 D(t)/V(t) D_0)$ is governed by the following SDE

$$d\chi(t) = \mu dt + \bar{\sigma} dW(t) + \bar{J} dN, \quad \chi(0) = 0,$$

Here $\mu = \nu\bar{\eta} + \sigma^2/2$, and jump size is positive.

Default occurs when χ hits the *upper* barrier $a = \ln(V_0/LD_0)$.

The survival probability of the firm is simply $Q_+(t,a)$.

A par credit spread $\Xi(T)$ for bonds maturing at time T can be expressed in terms of $Q_+(t,a)$, $0 \leq t \leq T$, as follows

$$\Xi(T) = (1-R)\left(\frac{1 - Z(T) Q(T,a) + \int_0^T Q(t,a) dZ(t)}{\int_0^T Z(t) Q(t,a) dt}\right).$$

Here $Z(t)$ is the risk-free discount factor, and R, $0 \leq R \leq 1$, is the recovery rate.

When $Z(t) = e^{-r^0 t}$ $\Xi(T)$ simplifies to

$$\Xi(T) = (1 - R)\left(\frac{1 - e^{-r^0 T} Q(T, a)}{\int_0^T e^{-r^0 t} Q(t, a)\, dt} - r^0\right).$$

We show the credit spread $\Xi(T)$ for realistic parameter values in Figure 6.

We emphasize that the main drawback of the conventional theory, namely the fact that $\Xi(T)$ is small when T is small, is not an issue when jumps are taken into account, so that randomization of the global recovery level L, as proposed by Finkelstein *et al.* (2001) is not needed.

Thus, we can hedge debt against puts on equity in a consistent way.

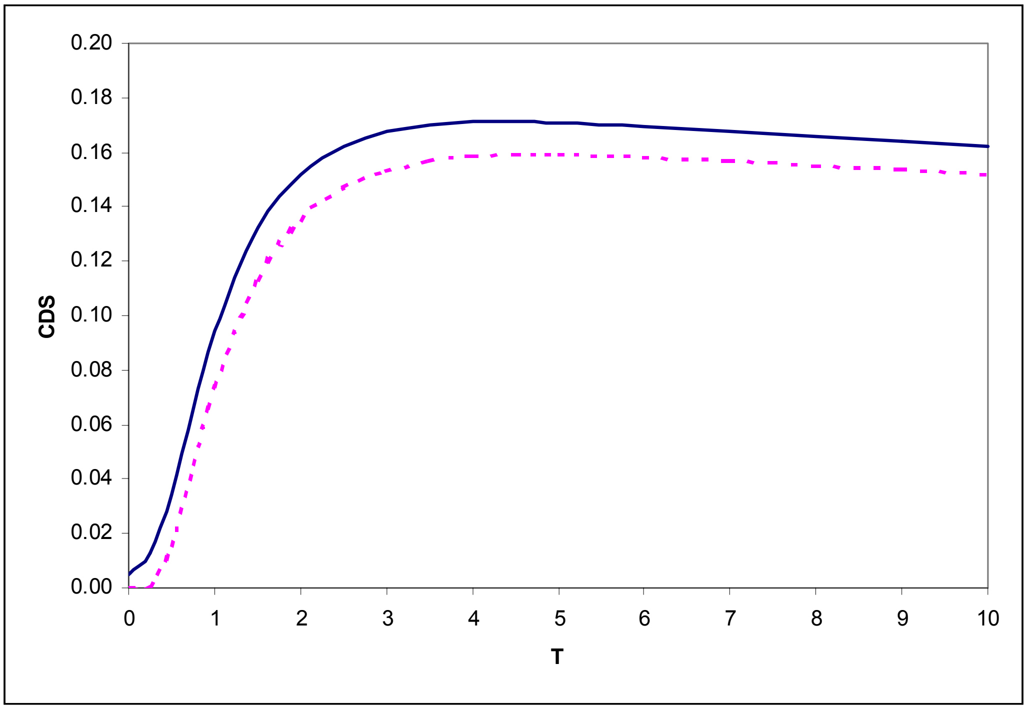

Figure 6: Credit default spread for a jump-diffusion and an equivalent diffusion.

8. CONCLUSIONS

1. We show how to use fluctuation identities for pricing path-dependent options.

2. Our derivation is complex but final formulas are simple and very general.

3. As a by-product we construct a new theory for computing credit default spreads.

4. The Lagrange inversion algorithm we use very efficiently solves other difficult problems such as pricing Asian, passport, and timer options.