MINERVA FOUNDATION LECTURES

Rama CONT (Oxford)

Rough Calculus

Tuesdays 4:10–5:25 PM

Room 622 Mathematics

2990 Broadway (117th Street)

The Ito calculus may be viewed as an extension of the Newton-Leibniz calculus to smooth functions of paths with non-zero quadratic variation. This analytical viewpoint is exploited to develop a calculus for smooth function(al)s of irregular paths with non-zero p-th variation for arbitrary p>1. Although this "rough calculus" is strictly pathwise in nature and does not involve any probabilistic ingredient, it is applicable to stochastic processes with irregular paths.

We illustrate the concepts and results of this theory in the setting of the Ito-Föllmer calculus for smooth function(al)s of paths with finite quadratic variation. We will then show how these results may be extended to the more general setting of smooth functionals of paths with non-zero p-th variation for arbitrary p>1, leading to a higher order Ito-type calculus. Finally, we will sketch some examples of applications to transport equations, optimal control and rough dynamics on manifolds.

I.   Ito calculus without probability

II.   Ito-Föllmer calculus for functionals of paths with finite quadratic variation.
Pathwise isometry and rough-smooth decompositions.

III.   Rough calculus for function(al)s of path with finite p-th variation.

IV.   The case of paths with fractional regularity (*)

V.   Transport of measures along rough trajectories.

VI.   Pathwise optimal control of dynamical systems driven by rough signals (*)

VII.   Rough dynamics on manifolds

*: if time permits

References:

Föllmer, H (1981) Calcul d'Ito sans probabilités. *Séminaire de probabilités de Strasbourg* 15 (1981): 143-150.

A Ananova, R Cont (2017) Pathwise integration with respect to paths of finite quadratic variation, Journal de Mathematiques Pures et Appliquees, Volume 107, No 6, June 2017, 737-757

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R Cont, N Perkowski (2019) Pathwise integration and change of variable formulas for continuous paths with arbitrary regularity, Transactions of the American Mathematical Society (Series B), Volume 6, 161-186.

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