Project title: The RSK correspondence and Pitman transforms for periodic random walks and polymers

Project leader: Evan Sorensen

Project description: The Robinson-Schendsted-Knuth (RSK) correspondence is a combinatorial bijection between the set of words on a finite alphabet and pairs of Young tableau. Understanding this bijection has led to significant advances in understanding the asymptotic behavior of many stochastic growth models, dating back to the turn of the century [BDJ99, Joh00]. A series of works in the early 2000s gave an interpretation of the RSK correspondence in terms of transformations of random walks [OY01, O'C03, OY02, BBO05, NY04]. A key related object is the Pitman transform [Pit75], which is a bijection on a suitable space of pairs of continuous functions (with several analogues for discrete functions). These developments have led to many probabilistic and algebraic consequences. A major breakthrough in the study of random growth models came in the construction of the directed landscape [DOV22], the conjectural (and proved for several special models) universal limit of growth models in the KPZ universality class. A key lemma proved in [DOV22] was an invariance of passage times under Pitman transformations of the weights.

Corwin [Cor21] recently proved a discrete tropical version of this invariance for discrete polymer models. In [CGS24], a periodic version of the discrete tropical Pitman transform was introduced and was shown to exhibit many analogues to known results for the Pitman transform on the full line. The goal of this project is to further investigate the structure of this periodic Pitman transform, both from an algebraic and probabilistic point of view. A particular emphasis will be given to comparison of structures in the full line, while demonstrating differences that occur in the periodic setting. On the algebraic side, the project seeks to prove an invariance result similar to that in [Cor21]. On the probabilistic side, we hope to develop an interpretation for the output of the periodic Pitman transform in terms of conditioned random walks, similar to that shown in [OY02]. At the beginning of the summer, we will study the relevant full-line objects and previous literature.

Prerequisites: Students should have taken linear algebra, a course in probability, at least one semester of undergradaute modern algebra, and at least one semester of undergraduate analysis.

References

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