Fermat's last theorem: syllabus

Fall 2024

Time and location: TBD
Overall course instructor: Prof. Miodrag Iovanov
Section instructor: Avi Zeff, email cbz2106@columbia.edu

Summary

The topic of this section is an overview of Fermat's last theorem, proven in the 1990s by Andrew Wiles and Richard Taylor. More precisely, Wiles and Taylor proved (a portion of) the Shimura–Taniyama–Weil conjecture, now often called the modularity theorem; most of the course will be concerned with understanding some of the concepts involved and how Fermat's last theorem follows from their result, together with a number of other results proven over the course of the 20th century. The proof of the modularity theorem is largely beyond the scope of the course, but we may discuss some of the ideas that go into it.

In setting the scene for the work of Wiles and Taylor, we will overview several different areas of modern number theory, including local and global fields and their Galois theory; elliptic functions and elliptic curves; and modular forms. We will not seek to become experts in any of these (vast) fields, but will try to learn just enough to see how they combine to relate Fermat's last theorem to the modularity theorem.

Prerequisites

Although it is not a strict requirement, we will frequently use the language of abstract algebra (groups, rings, ideals, fields...) and although we will review important results as needed it may be difficult to follow if you are not familiar with this language. At times we will also borrow tools from complex analysis; it is probably not necessary to have any deep familiarity with complex analysis, but at least a strong understanding of calculus will be important. Some familiarity with number theory may be useful, but is not required or especially expected.

On the other hand, we will discuss some topics which also appear in other classes, such as Galois theory, some pieces of algebraic number theory, and modular forms. No one topic will occupy too large a chunk of the seminar, but if you have already taken full courses on algebraic number theory and modular forms then you may find yourself bored.

For written assignments, the use of LaTeX is preferred but not required; we may also include some introduction to/resources for LaTeX in the class.

Textbook

The main reference for this class will be *Invitation to the Mathematics of Fermat–Wiles* by Yves Hellegouarch. We may at various points during the class turn to other sources, which will be distributed as needed.

Format

We meet once weekly for about two hours. Classes will mostly consist of student presentations, together with occasional lectures and discussion; depending on our pace and the number of students, each student will most likely present 2-3 times, though we may choose to instead have a larger number of shorter talks. In addition to preparing your talks, there will be some homework, focused on understanding, synthesizing, and effectively communicating the material, and a written final project on a topic of your choice. The final project may be done either individually or in small groups (depending on the size of the section), and we may choose to have some group presentations as well, although group presentations and projects will be expected to have commensurately larger scope. Finally, you will be expected to give your peers feedback on their presentations and other work.

Presentations

Each presentation should (barring alteration) be around 45-50 minutes, so a typical class session will consist of two presentations plus time for setup, questions, comments, and a brief break. You may choose to give either "slide talks" using a pre-made slide deck or "chalk talks" just on the blackboard; you may also choose to use handouts, if desired. We will discuss pros and cons of each option.

Before your presentation, you should meet with me to discuss what material you'll cover, sources, etc. (this can also be done by email if necessary), and give a practice presentation to me before your presentation to the class to receive feedback which you should then incorporate into your presentation.

Final projects

Your final project will be a paper on some topic related to the subject of the seminar, applying some of the tools we've learned. Its focus should be on investigating an idea beyond what we've strictly covered in class, and clearly communicating the results of your investigation to your peers. More precise details and expectations for the projects will appear later in the semester.

Grading

Grading will be based on the following:

- presentations;
- homework;
- final projects;
- peer feedback.

For the first three items, I am looking for correctness and quality of exposition; for peer feedback (that you will give your classmates on presentations, some homework, and final projects), I am looking for completion. You will receive feedback from both myself and your peers on each of the first three items, and do some self-assessment. You'll also have opportunities to revise errors on homework in response to feedback.

In addition to feedback, each assignment (presentations, homeworks, and the final project) will receive an overall mark of either E (excellent), S (satisfactory), or N (not yet satisfactory). Borderline work may receive "intermediate" marks, to be resolved to standard marks depending on e.g. future work or revisions. Your grade will be the highest row in the following table for which you've achieved *all* of the requirements:

Grade	**Presentations**	**Homeworks**	**Final project**
A	All at least S, at least one E	All at least S	E
B	All at least S	All at least S	At least S
C	At least one S	At most one N	At least S
D	At least one S	At least one S	No requirement

Plus or minus modifiers will be used for cases where you are almost at the next grade level or significantly above the requirements for one grade but not yet reaching the next one: for example, if you have all the requirements for an A except for an E on a presentation, you might get an A-; if you have all the requirements for a C but with only one N on homework as well as an E, you might get a C+. Work not meeting the requirements for a D will receive an F.

If you miss giving peer feedback without prior excuse, I will notify you and give you the chance to make it up; if it is not completed within a few days it will lower your grade by a modifier (A to A-, A- to B+, etc.) for every two missing feedbacks.

Course policies

Attendance

Attendance is a crucial part of the participation in the seminar. Students will spend more time as an audience member than giving their own talks. Students are expected to attend all meetings and, in the event of a virtual meeting, they are expected to have their cameras on. Absences must be excused in advance (barring exceptional circumstances). If you are unable to come to class in person but are available virtually, we may be able to arrange a Zoom option. If you regularly need to arrive late or leave early, you should arrange a way to still give full feedback with me.

Deadlines and extensions

Due to the peer evaluation process, it is very important to turn in homework on time. If you are absolutely unable to do so, please notify me in advance if at all possible, preferably with at least 24 hours' notice, and we may be able to work out an extension; you should expect unexcused or repeatedly late homework to count against you.

If you are scheduled to give a presentation and cannot (e.g. due to illness or another emergency), please give me as much notice as possible—preferably at least 48 hours, ideally more—and we will see if we can rearrange things at all.

Joining late

If you are joining the class after the first meeting, you will be expected to make up any homeworks you missed, but not peer feedback. You will need to give the same number of

presentations as everyone else, so they may be condensed into a shorter amount of time. After the first class meeting, it becomes more difficult, though not necessarily impossible, to rearrange planned talks to accommodate additional students, and further into the semester even more so, so I cannot guarantee that it will be possible to switch into the class later into the semester, but you are welcome to ask.

COVID-19 policies

Classes will be in person, but please do not come to class if you are feeling sick or test positive—illness is always a valid reason to miss class. It may be possible to stream classes on Zoom as needed.

Academic Honesty Policy

Please read (and follow) the Columbia University Undergraduate Guide to Academic Integrity. If you are ever unsure whether something is allowed, please ask me first—you will never be penalized for asking.

Accessibility and accommodations

Please let me know if there is anything I can do to make this course more accessible to you, or if aspects of the course are excluding you, and we can work together to develop strategies to improve the class. If you think you may need official accommodations, I encourage you to contact the Office of Disability Services for an accommodation letter.

Feedback

Please feel free to give me comments on the class at any time, via email or in person; if you are not comfortable doing either, you can give me anonymous feedback via this form.