Pre-class worksheet 1: limit laws

Calculus I, section 10

Due September 14, 2023 by 4:10 PM

Read the following text, and complete the problems below.

In class, we discussed limit laws. These give ways to split up limits in a variety of ways, under reasonable conditions: so long as $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, we have

$$\lim_{x\to a} (f(x)g(x)) = \left(\lim_{x\to a} f(x)\right) \cdot \left(\lim_{x\to a} g(x)\right),$$

$$\lim_{x\to a} (f(x)+g(x)) = \left(\lim_{x\to a} f(x)\right) \cdot \left(\lim_{x\to a} g(x)\right),$$

and if additionally $\lim_{x\to a} g(x) \neq 0$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}.$$

As a special case of the multiplication law, if $g(x) = c$ is constant then $\lim_{x\to a} c = c$, and so we get

$$\lim_{x\to a} cf(x) = c \lim_{x\to a} f(x).$$

Taking $c = -1$ and combining with the addition law gives a limit law for subtraction as well.

We could combine these laws in other ways to get similar statements. For example,

$$\lim_{x\to a} f(x)^2 = \left(\lim_{x\to a} f(x)\right)^2$$

if both sides exist, by the multiplication limit law. We could keep going like this to see that

$$\lim_{x\to a} f(x)^n = \left(\lim_{x\to a} f(x)\right)^n$$

for any n, so long as both sides exist.

There's one other very powerful limit law we haven't talked about: function composition. If $\lim_{x\to a} g(x)$ exists and is equal to some number L, then

$$\lim_{x\to a} f(g(x)) = \lim_{y\to L} f(y).$$

For example, to compute

$$\lim_{x\to 2} \log_2\left(\frac{x^2-4}{x-2}\right),$$

this limit law tells us that we can first compute

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)}{(x+2)}x-2 = \lim_{x\to 2}(x+2) = 4,$$

and then our limit is

$$\lim_{y\to 4} \log_2(y) = \log_2(4) = 2.$$

This lets us think about certain complicated limits piece-by-piece.

Problem 1. Suppose that $f(x)$ is some function that we know has the property that $\lim_{x\to 1} f(x) = 2$. What is $\lim_{x\to 1} \frac{1}{f(x)}$?

Problem 2. Consider the limit $\lim_{x\to\frac{\pi}{2}} \frac{1}{\tan(x)}$. Which of the following is correct?

(a) As $\lim_{x\to\frac{\pi}{2}} \tan(x)$ does not exist, the overall limit cannot exist either.

(b) As $\lim_{x\to\frac{\pi}{2}} \tan(x)$ does not exist, we cannot apply the limit law; the limit might exist or might not, we would need other methods to say.