First Exam

Modern Algebra II, Dave Bayer, October 5, 2010

Name: ______________________________

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[**1**] Define a maximal ideal, and give an example of a maximal ideal. Define a prime ideal, and give an example of a prime ideal. Give an example of three prime ideals $I \subset J \subset K$, each strictly contained in the next.

[**2**] Compute $5^{32} \bmod 77$.

[**3**] A message is represented as an integer $a \mod 57$. You receive the encrypted message $a^{11} \equiv 2 \mod 57$. What is a?

[**4**] Let A be a 2×2 matrix with entries in $\mathbb{R}$, satisfying the polynomial relation

$$(x-1)(x-3) \;=\; 0$$

Find a formula for A^n as a polynomial expression in A. What is $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^n$?

[5] Construct the finite field $\mathbb{F}_9$ as an extension of $\mathbb{F}_3 = \mathbb{Z}/3\mathbb{Z}$, by finding an irreducible polynomial of degree 2 with coefficients in $\mathbb{F}_3$. Find a generator of the multiplicative group $\mathbb{F}_9^*$ of nonzero elements of $\mathbb{F}_9$. Demonstrate that your choice is indeed a generator.