Exam 2

Modern Algebra II, Dave Bayer, November 6, 2008

Name: ____________________

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

[**1**] Define a primitive polynomial, for polynomials in $\mathbb{Z}[x]$. Prove *Gauss's lemma*: The product of two primitive polynomials is primitive.

[2] Define an irreducible element of an integral domain. Show that in a principal ideal domain, every element which is neither zero nor a unit is a product of irreducible elements.

[**3**] Make a list of small irreducible elements for the ring $\mathbb{Z}[\sqrt{-5}]$.

[4] Let $I = (30, 72, x^4 - 1)$ be an ideal in $\mathbb{Z}[x]$. Find a strictly ascending chain of ideals of $\mathbb{Z}[x]$

$$I \subset I_2 \subset \ldots \subset I_k$$

of maximal length.

[5] Find a strictly ascending chain of *prime* ideals of $\mathbb{Z}[x, y]$

$$I_1 \subset I_2 \subset \ldots \subset I_k$$

of maximal length.