Take-Home Exam 2

Modern Algebra II, Dave Bayer, October 28, 2008

Name: ______________________________

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

[**1**] Make a list of small primes for the Gaussian integers $\mathbb{Z}[i]$.

[2] Define a principal ideal domain. State the ascending chain condition. Show that a principal ideal domain satisfies the ascending chain condition.

[**3**] Define an irreducible element. Show that in a principal ideal domain, every element which is neither zero nor a unit is a product of irreducible elements.

[4] Define a unique factorization domain. Show that a principal ideal domain is a unique factorization domain.

[5] Let $I = (b^2 - ac, bc - ad)$ be an ideal in the polynomial ring $R = F[a, b, c, d]$ for F a field. Is the quotient ring R/I an integral domain? Can you give a geometric explanation for your answer?