Exam 2

Modern Algebra II, Dave Bayer, November 19, 2009

Name: ____________________

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[**1**] Reduce the matrix $A = \begin{bmatrix} 30 & 30 & 30 \\ 30 & 10 & 30 \\ 30 & 30 & 6 \end{bmatrix}$ to diagonal form by integer row and column operations.

[2] Define a principal ideal domain. State the ascending chain condition. Show that a principal ideal domain satisfies the ascending chain condition.

[3] Let $\mathsf{F} \subset \mathsf{K} \subset \mathsf{L}$ be fields. Prove that $[\mathsf{L} : \mathsf{F}] = [\mathsf{L} : \mathsf{K}]\,[\mathsf{K} : \mathsf{F}]$.

[**4**] Let $F \subset K \subset L$ be fields. Prove that if L is algebraic over K and K is algebraic over F, then L is algebraic over F.

[**5**] Prove the Hilbert Basis Theorem: If a ring R is noetherian, then so is the polynomial ring $R[x]$.