Additional Practice Problems for Exam 2

Modern Algebra II, Dave Bayer, November 12, 2009

Name: ______________________________

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] Prove that a principal ideal domain is a unique factorization domain.

[**2**] State and prove Gauss's Lemma.

[**3**] State and prove the Eisenstein Criterion.

[**4**] Prove the Hilbert Basis Theorem: If a ring R is noetherian, then so is the polynomial ring $R[x]$.

[5] Reduce the matrix $A = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 10 & 12 & 14 & 16 \end{bmatrix}$ to diagonal form by integer row and column operations.

[6] Let $F \subset K \subset L$ be fields. Prove that $[L : F] \;=\; [L : K]\,[K : F]$.

[7] Let $F \subset K \subset L$ be fields. Prove that if L is algebraic over K and K is algebraic over F, then L is algebraic over F.