Final Exam

Modern Algebra II, Dave Bayer, December 22, 2009

Name: ______________________________

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	[6] (6 pts)	TOTAL

Please work only one problem per sheet of paper; each problem will be graded separately. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

Write the label "**Exam 11**" at the top of every new sheet of paper that you use; this identifies your exam. Do not fold, crumple, or staple your pages; they need to go through an automatic sheet feeder without jamming.

[**1**] Let K be the splitting field over $\mathbb{Q}$ of the polynomial $x^2 - 2x + 3$. Find an element $a \in \mathbb{Q}$ such that $K = \mathbb{Q}(\sqrt{a})$.

Exam 11

[2] Let K be the splitting field over $\mathbb{Q}$ of the polynomial $x^3 + px + q$, where $p, q \in \mathbb{Q}$. When is the degree $[K : \mathbb{Q}] = 3$? When is the degree $[K : \mathbb{Q}] = 6$? Give an example of a polynomial for each case.

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[3] Let K be the splitting field over the finite field $\mathbb{F}_3$ of the polynomial $x^4 - 1$. What is the Galois group $G(K/\mathbb{F}_3)$?

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[4] Prove the *Eisenstein Criterion*: If $f(x) \in \mathbb{Z}[x]$ and p is a prime, such that the leading coefficient of $f(x)$ is not divisible by p, every other coefficient of $f(x)$ is divisible by p, but the constant term is not divisible by p^2, then $f(x)$ is irreducible in $\mathbb{Q}[x]$.

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[5] Let K be the splitting field over $\mathbb{Q}$ of the polynomial $x^5 - 81x + 3$. What is the Galois group $G(K/\mathbb{Q})$?

Exam 11

[**6**] Prove the *Primitive Element Theorem*: Let K be a finite extension of a field F of characteristic zero. There is an element $\alpha \in K$ such that $K = F(\alpha)$.