Exam 1

Combinatorics, Dave Bayer, October 4, 2011

Name: _____

[1] (5 pts)	[2] (5 pts)	[3] (5 pts)	[4] (5 pts)	[5] (5 pts)	[6] (5 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[**1**] How many integers in the sequence

$$1,\ 2,\ 3,\ 4,\ \ldots\ 718,\ 719,\ 720$$

are not divisible by 4, 5, or 6?

[**2**] Count the number of ways of making change for n cents using pennies and nickels. Give a table of small values, and a generating function. For example, there are two ways of making change for 8 cents, namely

$$1+1+1+1+1+1+1+1, \qquad 1+1+1+5.$$

[**3**] Count the number of words using the letters 2 and 3 whose letters add up to n. Give a table of small values, and a recurrence relation. For example, there are four words that add up to 8, namely

$$2222, \qquad 233, \qquad 323, \qquad 332.$$

[4] The Fibonacci sequence $\{F_n\}$ is defined by the recurrence relation

$$F_n = F_{n-1} + F_{n-2}$$

with initial values $F_0 = 0$ and $F_1 = 1$. Solve for the generating function

$$G = \sum_{n=0}^{\infty} F_n \, t^n$$

[5] Count the number of ways of making change for n cents using 5, 6, and 7 cent coins. Give your answer as a generating function. Of these, how many ways *do not contain* a way to make change for twelve cents? For example,

$$5+5+5+5+5+5, \qquad 6+6+6+6+6$$

are two of the possible ways to make change for 30 cents, but one of them contains a way to make change for 12 cents.

[**6**] Approximately what fraction of all permutations of n letters have no fixed points? One fixed point? Two fixed points? If you keep going for k fixed points for each k, do your answers add up to one?