Exam 2

Combinatorics, Dave Bayer, November 17, 2011

Name: ______________________________

[1] (6 pts)	[2] (6 pts)	[3] (6 pts)	[4] (6 pts)	[5] (6 pts)	TOTAL

Please work only one problem per page, starting with the pages provided. Clearly label your answer. If a problem continues on a new page, clearly state this fact on both the old and the new pages.

[1] How many ways can a four term product be fully parenthesized? For example, $((ab)c)d$ and $(ab)(cd)$ are two different ways to multiply together $abcd$. Relate this count to binary trees, and draw the trees.

[2] A labeled tree on nine vertices has the Prüfer sequence 2,1,3,1,4,1,5. Draw the tree.

[3] How many different necklaces can be made from five beads, if three colors of beads are available, taking into account symmetry? (A necklace can be flipped and rotated.)

Of these necklaces, how many use all three colors?

[4] Prove *Burnside's lemma*: Let G be a finite group that acts on a finite set X. If X^{g} denotes the set of elements of X that are fixed by an element $\mathsf{g} \in \mathsf{G}$, then the number of orbits of this action is

$$\frac{1}{|\mathsf{G}|} \sum_{\mathsf{g} \in \mathsf{G}} |\mathsf{X}^{\mathsf{g}}|$$

[5] Prove the *five color theorem* for planar graphs: Every planar graph has a proper vertex coloring using only five colors. (You may assume that every planar graph has a vertex of degree five or less.)