

Exam 2

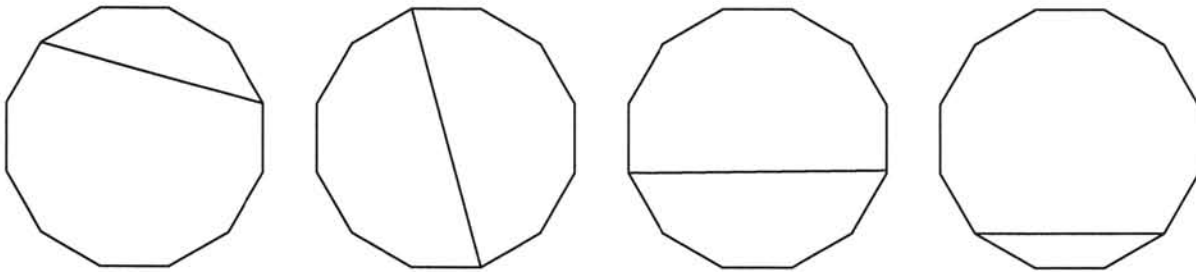
Combinatorics, Dave Bayer, March 6, 2014

Name: Solutions Uni: _____

[1]	[2]	[3]	[4]	[5]	Total

If you need more than one page for a problem, clearly indicate on each page where to look next for your work.

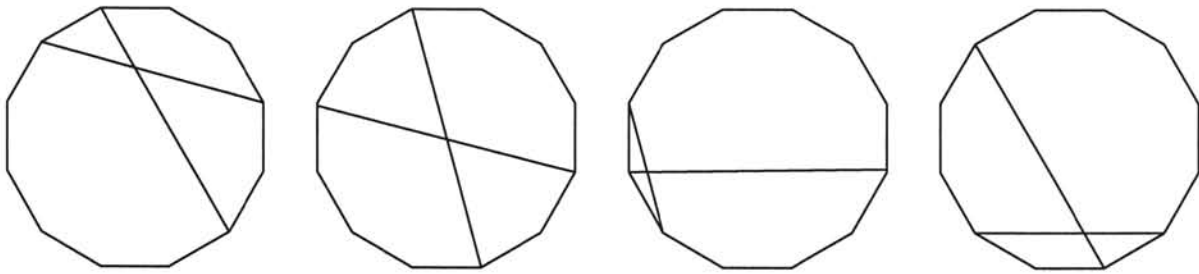
[1] How many ways can a 12-gon be dissected into two pieces, neither of which is a triangle?



$\binom{12}{2}$ pairs - 12 outside edges - 12 triangles

$$\frac{12 \cdot 11}{2 \cdot 1} - 2 \cdot 12 = \frac{12(11-4)}{2} = 6 \cdot 7 = \boxed{42}$$

[2] How many ways can one draw two crossing interior edges, inside a 12-gon?



Every 4 element subset of vertices yields one cross



$$\binom{12}{4} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = 11 \cdot 5 \cdot 9 = 11 \cdot 45$$

$$= 450 + 45$$

$$= \boxed{495}$$

[3] How many standard Young tableaux are there of the following shape?

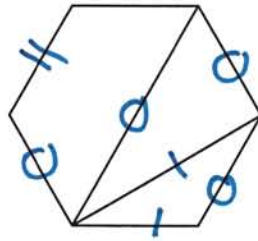
5	4	2
4	3	1
2	1	

hook lengths

$$\frac{\cancel{8} \cdot \cancel{7} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{1}} = 7 \cdot 6 = \boxed{42}$$

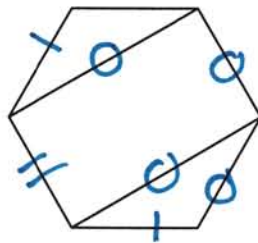
$$\frac{n!}{\prod \text{hook lengths}}$$

[4] Translate each of the following hexagon dissections into standard Young tableaux.



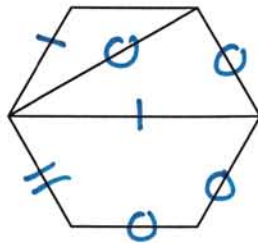
1	2	3
4	6	
5	7	

$\parallel \underbrace{\odot \odot} \mid \mid \underbrace{\odot \odot}$



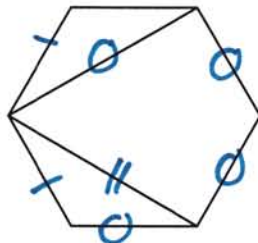
1	2	6
3	4	
5	7	

$\mid \underbrace{\odot} \parallel \underbrace{\odot} \mid \odot \odot$



1	2	7
3	5	
4	6	

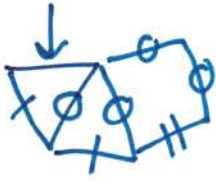
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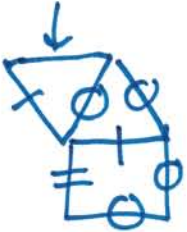
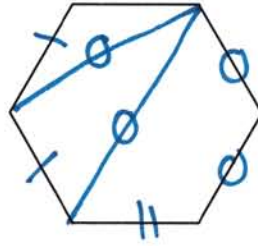
1	2	6
3	5	
4	7	

$\mid \underbrace{\odot} \parallel \mid \underbrace{\odot} \odot \odot$

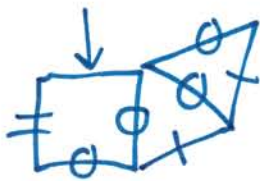
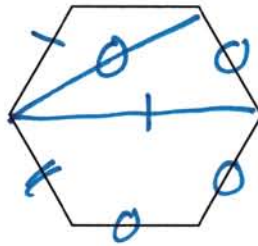
[5] Translate each of the following standard Young tableaux into hexagon dissections.



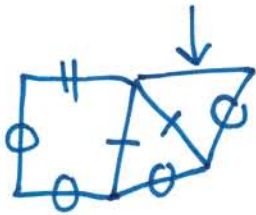
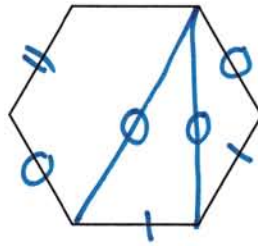
1	2	7
3	4	
5	6	



1	2	7
3	5	
4	6	



1	2	3
4	5	
6	7	



1	4	7
2	5	
3	6	

