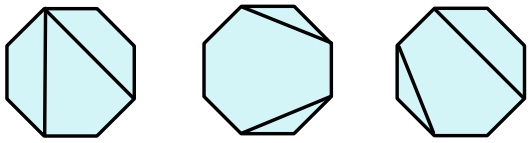
Final Exam

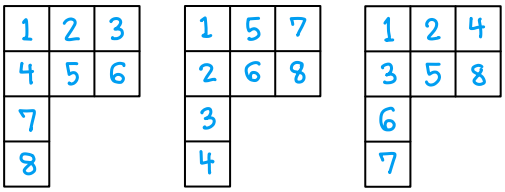
Combinatorics, Dave Bayer, April 20-23, 2021

To receive full credit for correct answers, please show all work.

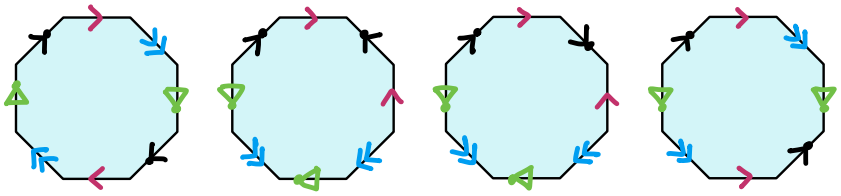
[1] How many ways can we dissect an octagon using 2 cuts? Provide a check of your answer.
(You may solve the problem two different ways, or classify the possibilities, or draw every possibility.)



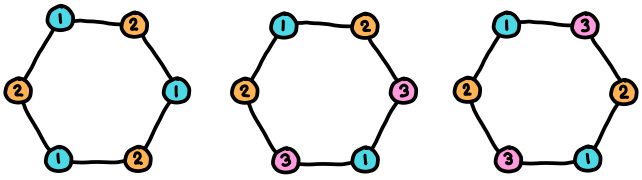
[2] For each of the following Young tableaux, find the dissection of an n-gon given by Stanley's correspondence.



[3] Identify each of the following surfaces from their gluing diagrams, computing their Euler characteristic and deciding whether or not they are orientable. Which two surfaces are homeomorphic (topologically equivalent)?



[4] How many ways can we properly color the vertices of a hexagon using n colors, up to rotational symmetry? Confirm your answer by drawing each of the possibilities for $n = 3$.
(For a proper coloring, adjacent vertices have distinct colors. You need not use every color.)



[5] How many ways can we dissect an octagon using 4 cuts, up to dihedral (rotations and flips) symmetry? Confirm your answer by drawing each of the possibilities. Which patterns are not chiral?