## Final Exam

Combinatorics, Dave Bayer, April 20-23, 2021
To receive full credit for correct answers, please show all work.
[1] How many ways can we dissect an octagon using 2 cuts? Provide a check of your answer.
(You may solve the problem two different ways, or classify the possibilities, or draw every possibility.)

[2] For each of the following Young tableaux, find the dissection of an n-gon given by Stanley's correspondence.

| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 5 | 6 |
| 7 |  |  |
| 8 |  |  |
|  |  |  |


| 1 | 5 | 7 |
| :--- | :--- | :--- |
| 2 | 6 | 8 |
| 3 |  |  |
| 4 |  |  |
|  |  |  |


| 1 | 2 | 4 |
| :--- | :--- | :--- |
| 3 | 5 | 8 |
| 6 |  |  |
| 7 |  |  |
|  |  |  |

[3] Identify each of the following surfaces from their gluing diagrams, computing their Euler characteristic and deciding whether or not they are orientable. Which two surfaces are homeomorphic (topologically equivalent)?

[4] How many ways can we properly color the vertices of a hexagon using $n$ colors, up to rotational symmetry? Confirm your answer by drawing each of the possibilities for $n=3$.
(For a proper coloring, adjacent vertices have distinct colors. You need not use every color.)



[5] How many ways can we dissect an octagon using 4 cuts, up to dihedral (rotations and flips) symmetry? Confirm your answer by drawing each of the possibilities. Which patterns are not chiral?


