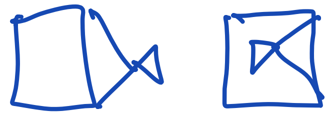
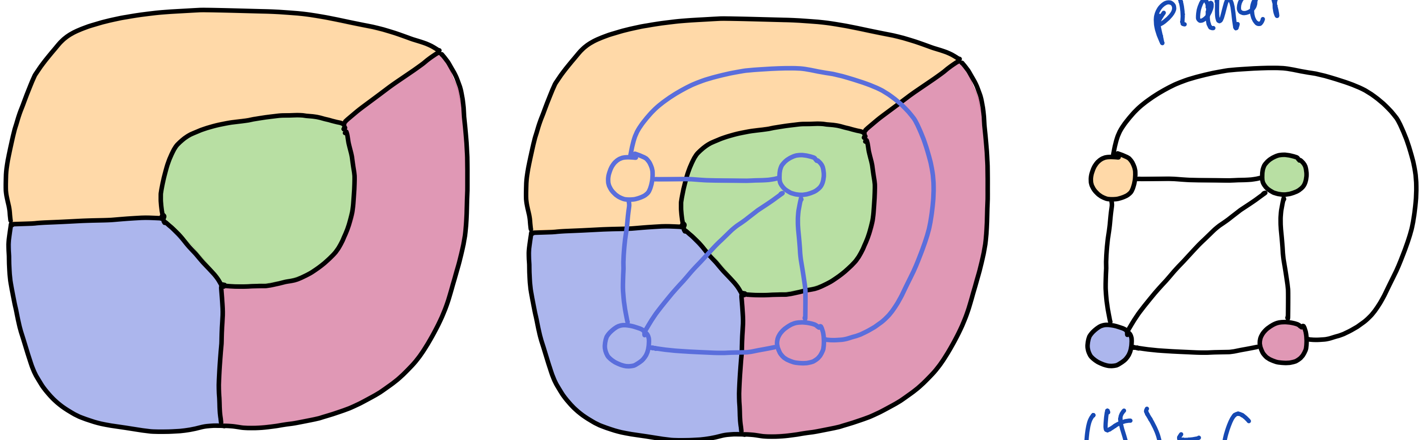
April 8 Graph Colorings



plane
planar



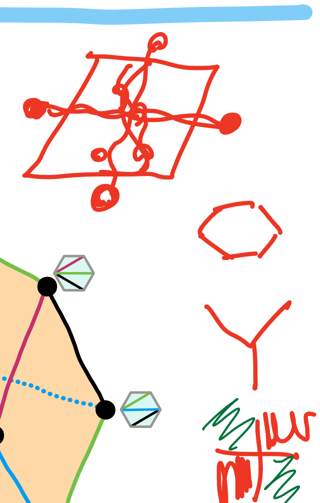
$\binom{4}{2} = 6$

A planar map can require 4 colors

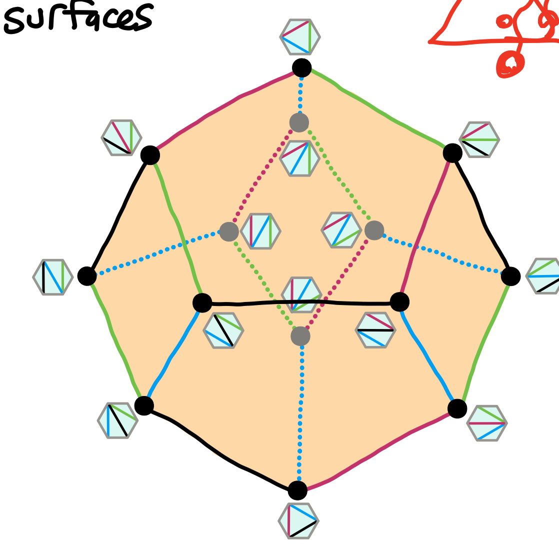
6 colors – easy to prove } we'll do
5 colors – harder
4 colors – very difficult, still no proof easily understood

Euler characteristic $\chi = v - e + f$

Invariant of simplicial, cellular surfaces

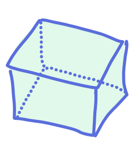


$\chi =$ # vertices
− # edges
+ # faces



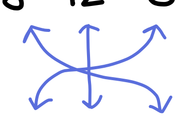
Associahedron

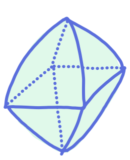
$\chi = 2 = 14 - 21 + 9$



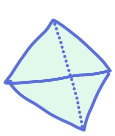
v e f

$\chi = 2 = 8 - 12 + 6$



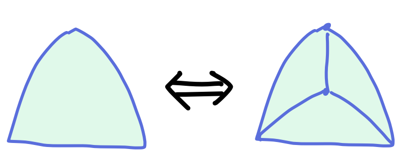


$\chi = 2 = 6 - 12 + 8$



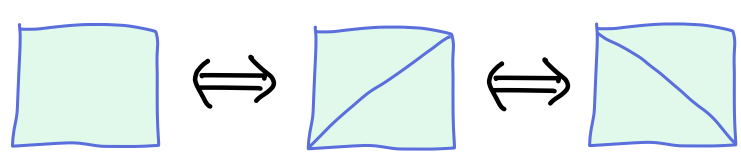
$\chi = 2 = 4 - 6 + 4$

$\chi = 2$ for any topological sphere (genus 0)



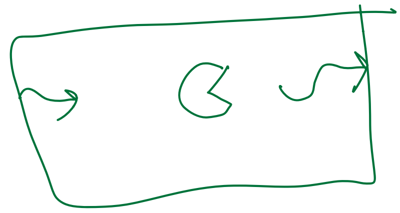
v e f

Δ 1−3+2 = 0



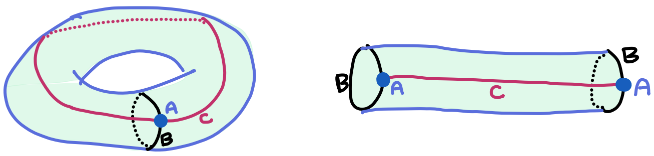
v e f

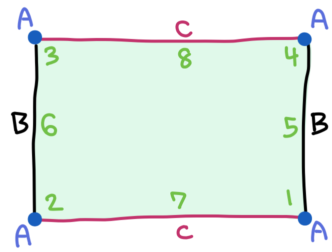
Δ 1−1 = 0

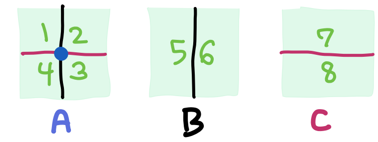


Torus

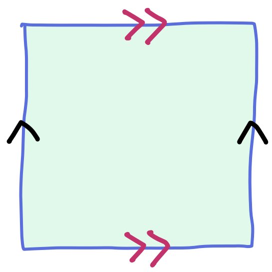
(genus 1)



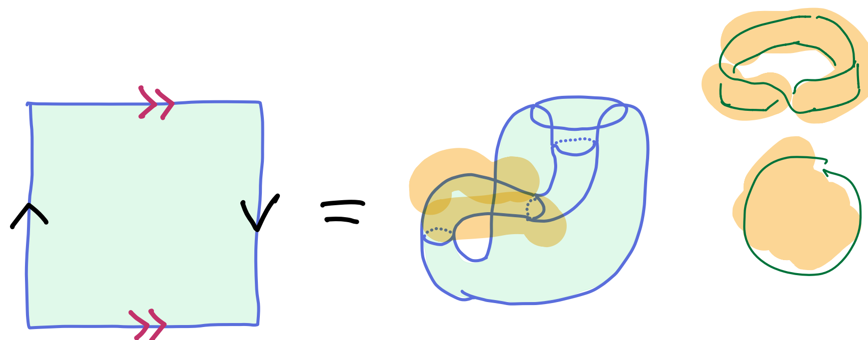




$$\chi = 0 = \underset{v}{1} - \underset{e}{2} + \underset{f}{1}$$

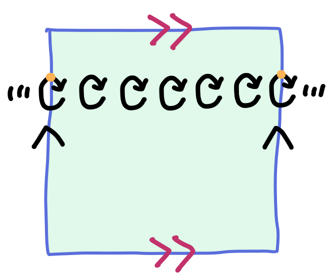


Torus

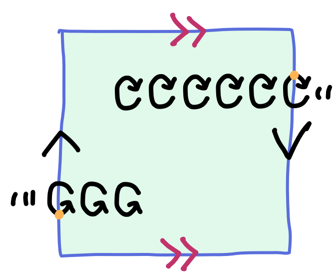


Klein bottle

(same χ)

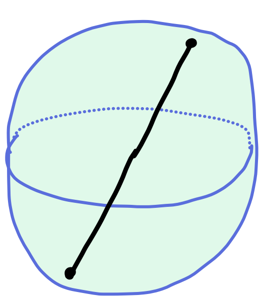


orientable

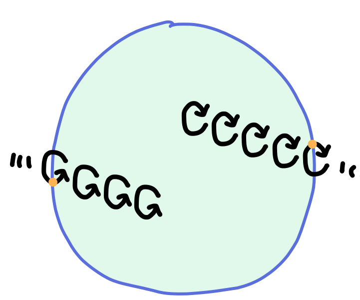


orientation-reversing path
⇒ not orientable

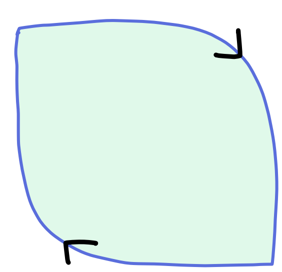
Projective plane



positions of a stick
if we can't tell ends apart

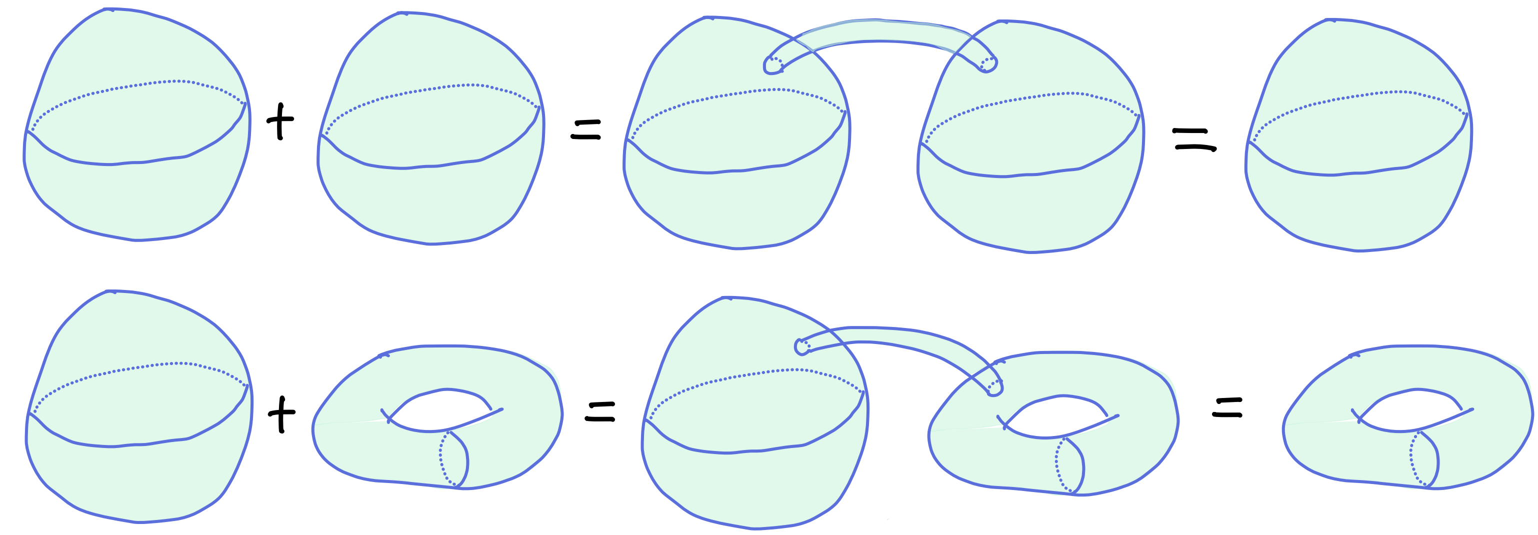


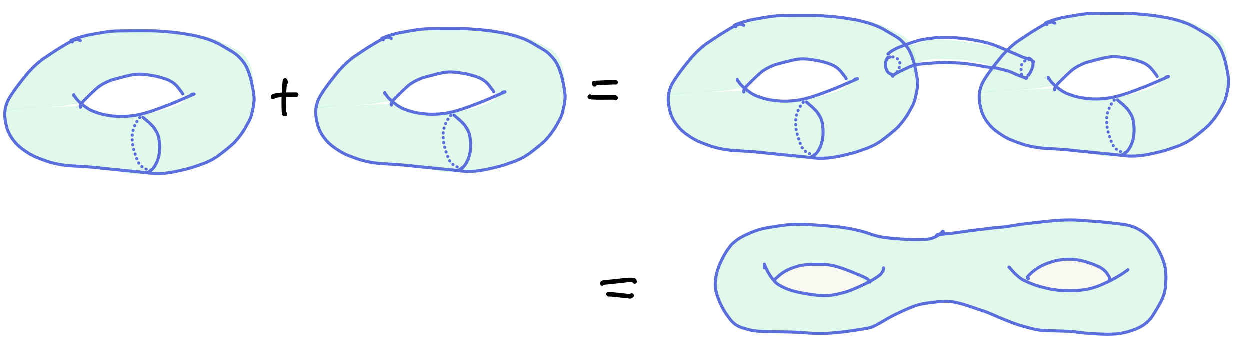
orientation-reversing path
⇒ not orientable



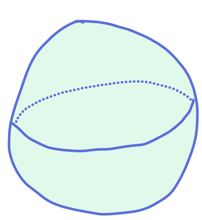
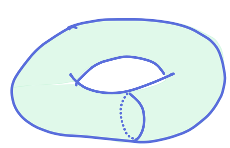
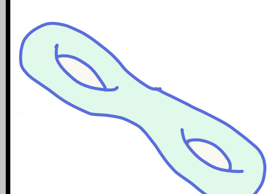
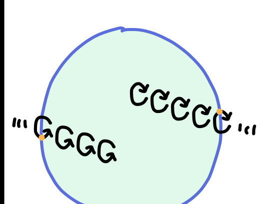
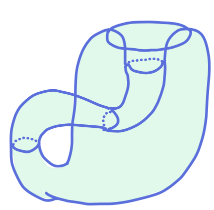
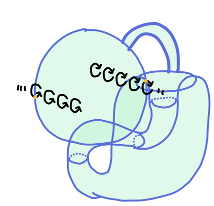
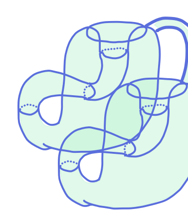
$$\chi = 1 = \underset{v}{1} - \underset{e}{1} + \underset{f}{1}$$

Surgery : Two surfaces can be "added" by connecting with a tube

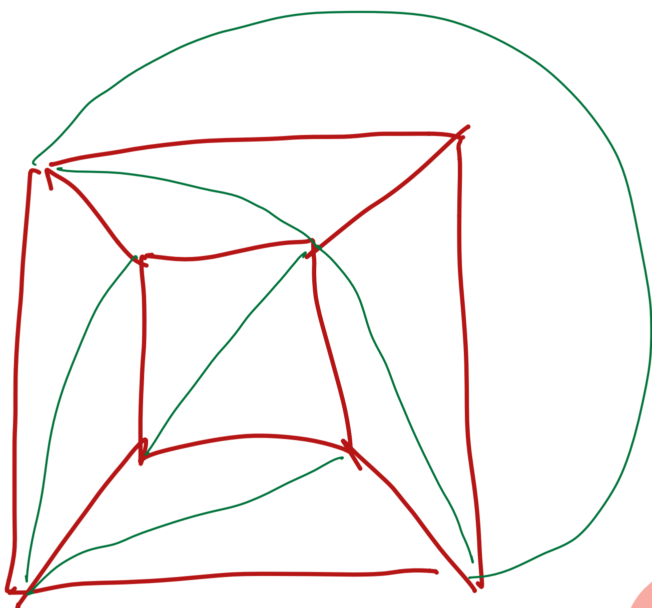




genus 2

	$\chi=2$	$\chi=1$	$\chi=0$	$\chi=-1$	$\chi=-2$	
orientable						'''
non-orientable						'''
	0	1	2	3	4	

complexity



$\chi = 2$

$v - e + f = 2$

e, f

$2e = 3f$