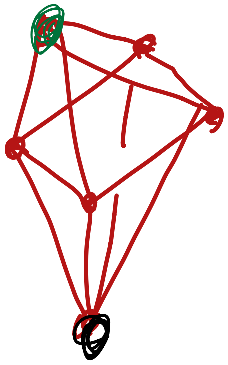
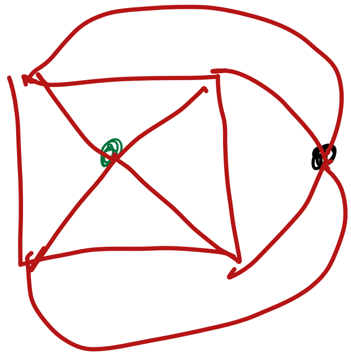
office Hours





reduce to triangulations

$\begin{cases} \text{adding edges doesn't hurt} \\ \text{triangulations easier.} \end{cases}$

tetrahedron $= 4\ \Delta$'s

$= 4 \cdot 3 = 12$ edge sides

$= 6$ edges

for any sphere

$\chi = v - e + f = 2$

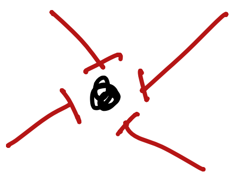
$3f = 2e$

$3 \cdot 4 \qquad 2 \cdot 6$

$12 \qquad 12$

each vertex meets many half edges

deg 3



deg 4

half edges in all

$= 2e$

$d =$ average number of edges meeting a vertex

$d = (\#\text{half edges in all}) / V$

$dV = \#\text{ half edges in all } = 2e$

we end up with

$$\begin{cases} v - e + f = 2 \\ 3f = 2e \\ dv = 2e \end{cases}$$

$\therefore dv = 3f$

$\Downarrow$

$e = dv/2$

$f = dv/3$

want to show $d < 6$

plug into first eqn

$$v - e + f = 2$$

$$v - dv/2 + dv/3 = 2$$

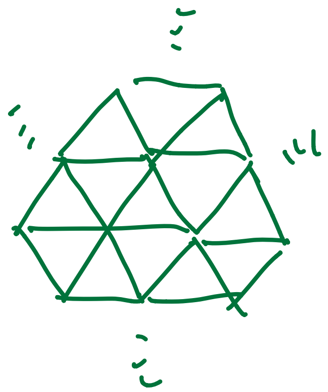
$$v(1 - d/2 + d/3) = 2$$

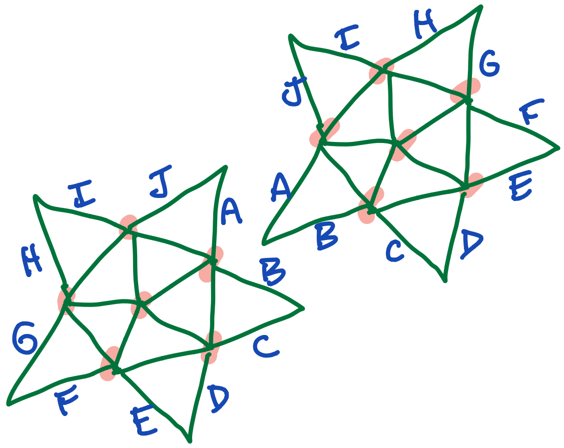
$$v(\underbrace{1 - d/6}_{>0}) = 2$$

$$1 - d/6 > 0$$

$$1 > d/6$$

$$6 > d$$



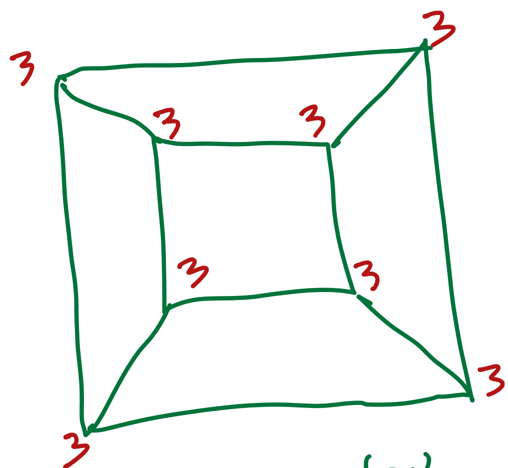


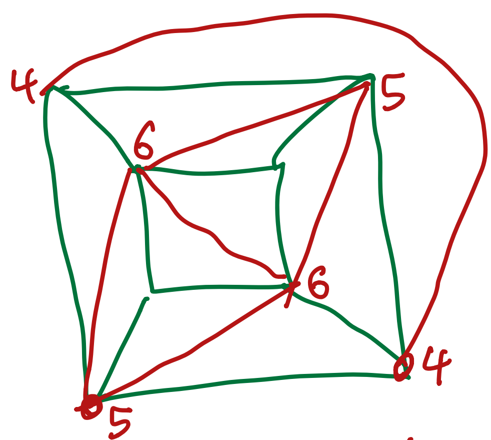
stick together
icosahedron
2D

Goal: find a vertex with as few neighbors as possible.

~~Best~~ we can do is 5 is general

We don't assume triangles. But don't hurt.





every vertex has 3 neighbors

now triangles harder
still the same vertices have deg < 6
easier to prove.

m = average number of sides of a face

$m \geq 3$

$$v - e + f = 2$$

$$mf = 2e \qquad f = dv/m$$

$$dv = 2e \qquad e = dv/2$$

$$v - dv/2 + dv/m = 2$$

$$v\,(\underbrace{1 - dv/2 + dv/m})= 2$$

$$\frac{dv}{2} - \frac{dv}{m} = \frac{m\,dv - 2dv}{2m}$$

$$v\left(1 - \frac{m-2}{2m} d\right) = 2$$

$$1 > \frac{m-2}{2m} d$$

$$\frac{2m}{m-2} > d$$

$$m \geq 3$$

$m = 3$

$$\frac{6}{1} > d$$

$m = 4$

$$\frac{2m}{m-2} = \frac{8}{2} = 4 > d$$

$$\frac{2m}{m-2} = 5$$

$$2m = 5(m-2)$$

$$2m = 5m - 10$$

$$10 = 3m$$

$$m = 10/3 \Rightarrow 5 > d$$

$\Rightarrow$ 4CT easy if average face has $\geq 10/3$ sides?

closer to 3 = nearly triangulation

of course, pulling a vertex could lower m, ruining recursion?