Office Hours Tue Jan 19

$\mathbb{Z}/n\mathbb{Z}$ notation for "integers mod n"

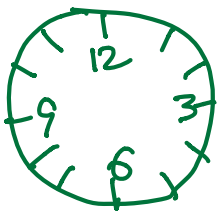
Equivalence relation ($\frac{4}{6} \simeq \frac{2}{3}$ used to define fractions)

Here any number $\simeq$ its remainder under division by n

$\mathbb{Z}$: ... -7 -6 -5 -4 -3 -2 -1 **0 1 2 3 4** 5 6 ...

$n=5$: ... 3 4 0 1 2 3 4 **0 1 2 3 4** 0 1 ...

idea $5 \simeq 0$



hours work mod 12

n prime $\Leftrightarrow$ field like $\mathbb{R}, \mathbb{C}$

$n=5$

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

*	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

$n=4$

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

*	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

mod 9 = "casting out nines"

check addition or check multiplication

$254 = 792 - 378 - 280 + 120$

From class

$792 \equiv 0$
$120 \equiv 3$
$912 \equiv 3$

$-\ 378 \equiv 0$
$280 \equiv 1$
$658 \equiv 19 \equiv 1$

$=$

$912 \equiv 3$
$-658 \equiv 1$
$254 \equiv 2$

$10 = 9+1 = 1 \mod 9$
$100 = 99+1 = 1 \mod 9$

so $912 = 9\cdot100 + 1\cdot10 + 2$
$\Downarrow \quad \Downarrow \quad \Downarrow \mod 9$
$9\cdot1 + 1\cdot1 + 2$
$= 9+1+2 = 12$
$= 1+2 = 3$

same mod 9

long way

$$\begin{array}{r} 101 \\ 9\overline{)912} \\ -900 \\ \hline 12 \\ -9 \\ \hline 3 \end{array}$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

2 A
3 B
5 C

at least this property

$$\emptyset - A - B - C + AB + AC + BC - ABC$$

$$60 - 30 - 20 - 12 + 10 + 6 + 4 - 2$$

$$60/2 \quad 60/3 \quad 60/5 \qquad 60/2\cdot3 \quad 60/2\cdot5 \quad 60/3\cdot5 \qquad 60/2\cdot3\cdot5$$

$$16 = \underbrace{-2}_{60/2,\ 60/3,\ 60/5} \quad \underbrace{+20}_{60/2\cdot3,\ 60/2\cdot5,\ 60/3\cdot5} \quad -2$$