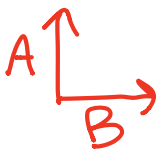
Tue Jan 19

1	6	18	41	41	67	132	254
1	5	12	23		26	65	122
1	4	7	11	17	26	39	57
1	3	3	4	6	9	13	18
1	2		1	2	3	4	5
1	1	1	1	1	1	1	1

end

start



Casting out nines?

$\mathbb{Z}/n\mathbb{Z}$ integers mod n $n \equiv 0$

$\mathbb{Z}/3\mathbb{Z}$

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

*	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

mod 9 $10 = 1+9 = 1$

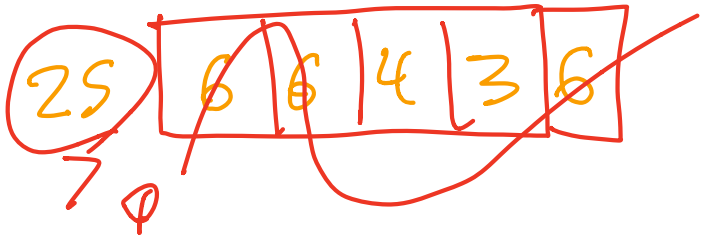
$190 = 1+99 = 1+9\cdot 11 = 1$

$1356 = 1+3+5+6 = 6$

1	6	0	5	5	4	6	2
1	5	3	5		8	2	5
1	4	7	2	8	8	3	3
1	3	3	4	6	0	4	0
1	2		1	2	3	4	5
1	1	1	1	1	1	1	1

end

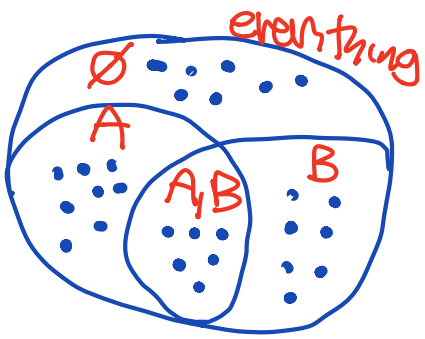
start

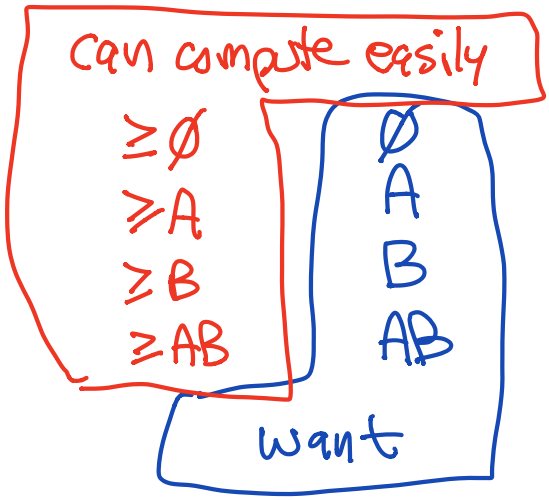


end

1	6	18	41	41	67	132	254
1	5	12	23	**B**	26	65	122
1	4	7	11	17	26	39	57
1	3	3	4	6	9	13	18
1	2	**A**	1	2	3	4	5
1	1	1	1	1	1	1	1

start



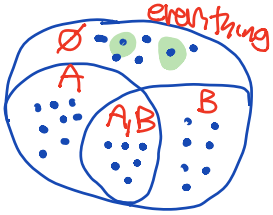


$$\geq\emptyset = \emptyset + A + B + AB$$
$$\geq A = A + AB$$
$$\geq B = B + AB$$
$$\geq AB = AB$$

$$\begin{bmatrix} \geq\emptyset \\ \geq A \\ \geq B \\ \geq AB \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \emptyset \\ A \\ B \\ AB \end{bmatrix}$$

$$\emptyset = \geq\emptyset - \geq A - \geq B + \geq AB$$
$$A = \geq A - \geq AB$$
$$B = \geq B - \geq AB$$
$$AB = \geq AB$$

$$\begin{bmatrix} \emptyset \\ A \\ B \\ AB \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \geq\emptyset \\ \geq A \\ \geq B \\ \geq AB \end{bmatrix}$$



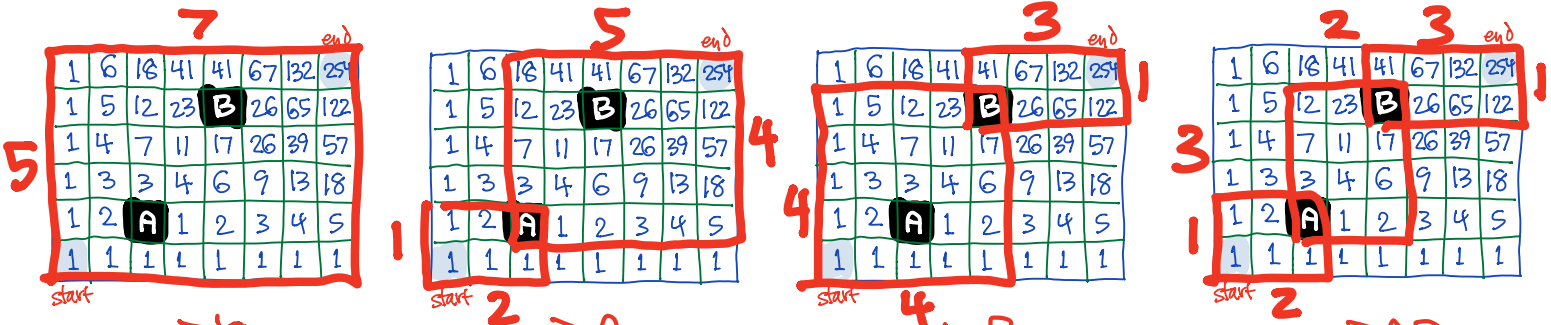
	$\geq\emptyset$	$-\geq A$	$-\geq B$	$+\geq AB$	
$\emptyset$	1				1
A	1	-1			0
B	1		-1		0
AB	1	-1	-1	1	0

1
1 1
1 2 1
1 -3 +3 -1 = 0
1 4 6 4 1

ABC

1 | -1 -1 -1 | 1 1 1 | -1

$(1-1)^n = 0$



$$\binom{12}{5} - \binom{3}{1}\binom{9}{4} - \binom{8}{4}\binom{4}{1} + \binom{3}{1}\binom{5}{3}\binom{4}{1}$$

$$\frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5\cdot 4\cdot 3\cdot 2\cdot 1} - \frac{3\cdot 9\cdot 8\cdot 7\cdot 6}{1\cdot 4\cdot 3\cdot 2\cdot 1} - \frac{8\cdot 7\cdot 6\cdot 5\cdot 4}{4\cdot 3\cdot 2\cdot 1\cdot 1} + \frac{3\cdot 5\cdot 4\cdot 3\cdot 2}{1\cdot 3\cdot 2\cdot 1\cdot 1}$$

$12\cdot 66$

660

132

$9\cdot 42$

$420 - 42$

$$254 = 792 - 378 - 280 + 120$$

$254 = 245 + 4 = 2$ (mod 3)

casting out nines: $0 \quad 0 \quad -1 \quad +3 \quad = 2$

Exercise: How many integers in 1..60 are not divisible by 2, 3, or 5?

A B C

Hat check problem

How many permutations are fixed point free

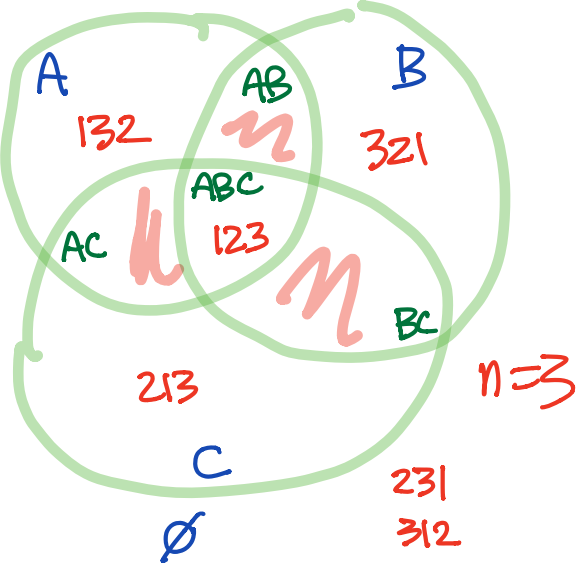
n=1: 1

n=2: 12, 21

n=3:

123	ABC
132	A
213	C
231	∅
312	∅
321	B

ABC



2134

AB CD

work with arbitrary n

$$\geq 0 \;\Big|\; -\geq A - \geq B - \geq C \;\Big|\; +\geq AB + \geq AC + \geq BC \;\Big|\; -\geq ABC$$

$$n! \;\Big|\; -\binom{n}{1}(n-1)! \;\Big|\; +\binom{n}{2}(n-2)! \;\Big|\; -\binom{n}{3}(n-3)!$$

$\binom{n}{2} = \frac{n(n-1)}{2 \cdot 1}$ ↖

111

$$n! \quad - \frac{n!}{1} \quad + \frac{n!}{2!} \quad - \frac{n!}{3!} + \cdots$$

$$n!\left(1 - 1 + \frac{1}{2} - \frac{1}{6} \cdots\right) = \left[n!/e\right] \quad 6/27$$

$$e^x = 1 + x + x^2/2! + x^3/3!$$

$$e^{-1} = 1 - 1 + 1/2 - 1/6 + 1/24 \cdots$$