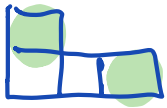
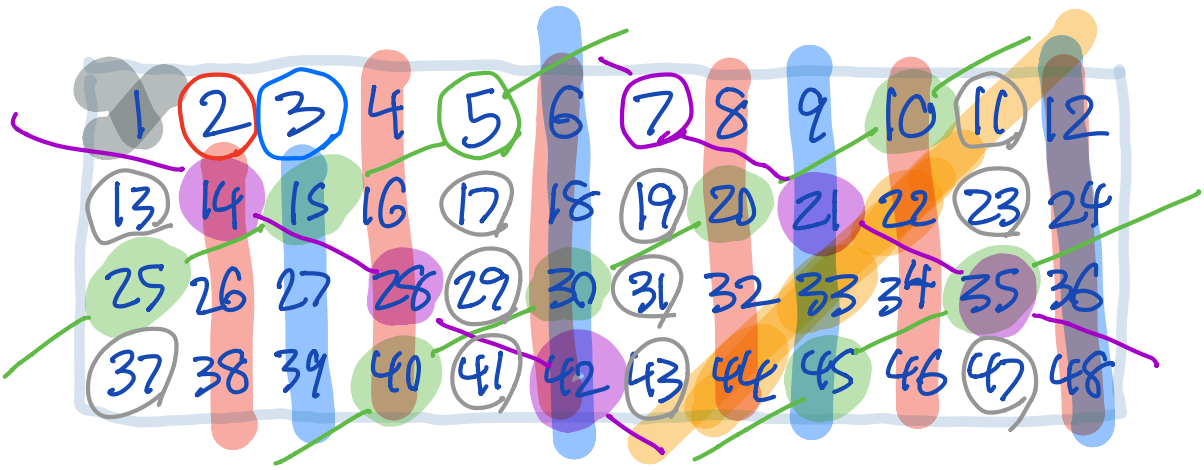
Thurs Jan 21



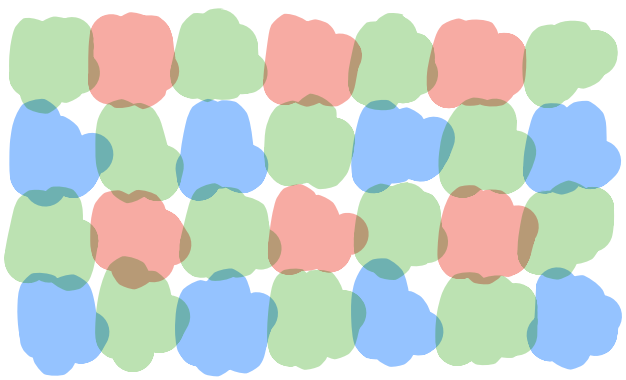
Prime sieve



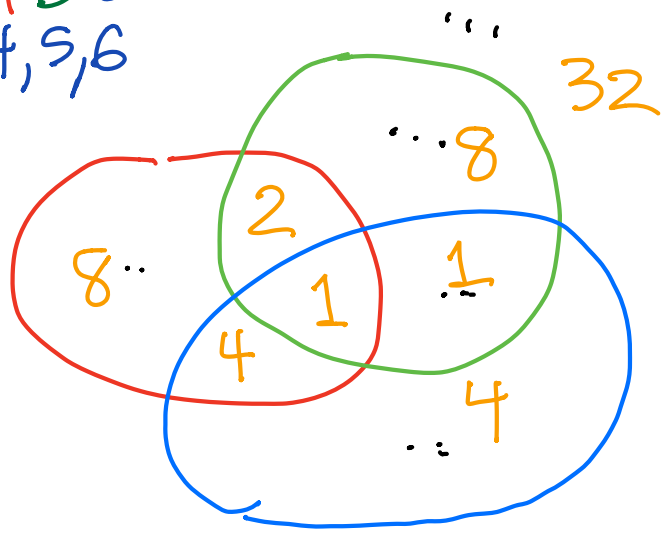
Relatively prime

divisibility by A B C
4, 5, 6

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60



A B C
4, 5, 6



$\geq A \quad 15 = 60/4$

$\geq B \quad 12 = 60/5$

$\geq C \quad 10 = 60/6$

$\geq AB\ 3 = 60/20$ $20 = 4 \cdot 5$

$\geq AC\ 5 = 60/12$ $12 = 4 \cdot 3$ lcm (4,6)

$\geq BC\ 2 = 60/30$ $30 = 5 \cdot 6$

$\geq ABC\ 1 = 60/60$ $60 = \text{lcm}(4,5,6)$

$$\phi = 60 - (15+12+10) + (3+5+2) - 1$$

$$60 - 37 + 10 - 1 = \boxed{32}$$

$$60\left(1 - 1/4 - 1/5 - 1/6 + 1/20 + 1/12 + 1/30 - 1/60\right)$$

1..30 not divisible by 2 or 3

$30 - 15 - 10 + 5 = 10$

$0! = 1$

$= 30(1 - 1/2 - 1/3 + 1/6)$

$30(1-1/2)(1-1/3) = 30 \cdot \frac{1}{2} \cdot \frac{2}{3} = 10$

Euler's totient function $\varphi(n)$ "phi"

integers $< n$, relatively prime to n

(include 1)

$\varphi(20) = 1, 3, 7, 9, 11, 13, 17, 19$

$20 = 2 \cdot 2 \cdot 5$

2, 5

A B

$(1-A)(1-B)$

$= 1 - A - B + AB$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

$20(1-1/2)(1-1/5)$

$2 \cdot 20 \cdot \frac{1}{2} \cdot \frac{4}{5} = 40/5 = 8$ ✓

primes for m, n

p, q prime factors of m

r, s prime factors of n

if m, n relatively prime

$$\varphi(m) * \varphi(n) = \varphi(mn)$$

$$m\left(1-\tfrac{1}{p}\right)\left(1-\tfrac{1}{q}\right) * n\left(1-\tfrac{1}{r}\right)\left(1-\tfrac{1}{s}\right) = mn\left(1-\tfrac{1}{p}\right)\left(1-\tfrac{1}{q}\right)\left(1-\tfrac{1}{r}\right)\left(1-\tfrac{1}{s}\right)$$

(after class)

$12 = 2\cdot 2\cdot 3$ $\quad 5 = 5$ $\quad 60 = 2\cdot 2\cdot 3\cdot 5$

2
3
5

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

$\varphi(12)$
$\varphi(5) = 4$

$\varphi(5) = 5(1-1/5) = 5\cdot 4/5 = 4$ ✓ $\quad$ 1 2 3 4 5

$\varphi(12) = 12(1-1/2)(1-1/3) = 12\cdot \tfrac{1}{2}\cdot \tfrac{2}{3} = 4$

$\varphi(60) = 4\cdot 4 = 16$

1	2	3	4	5	6
7	8	9	10	11	12

$p_1, \ldots, p_j$ prime factors of m

$q_1, \ldots, q_k$ prime factor of n

$$\varphi(m) = m \prod_{i=1}^{j}\left(1-1/p_i\right)$$

$$\varphi(n) = n \prod_{i=1}^{k}\left(1-1/q_i\right)$$

$$\varphi(mn) = mn \prod_{i=1}^{j}\left(1-1/p_i\right)\prod_{i=1}^{k}\left(1-1/q_i\right)$$