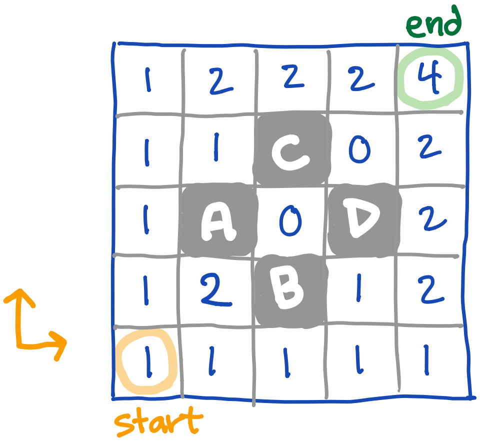
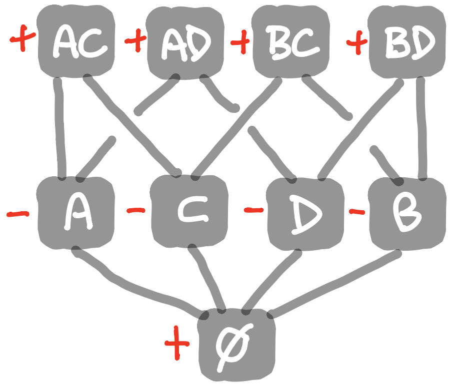
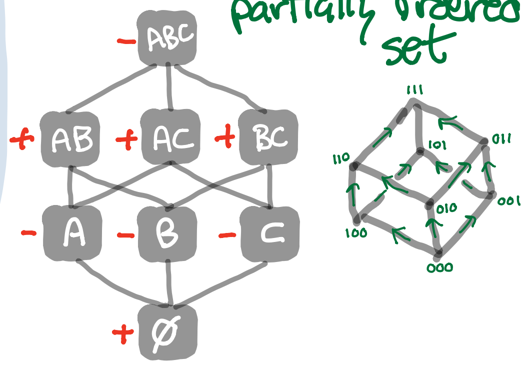
Jan 26 | Möbius inversion | poset





partially ordered set



(Special case: Inclusion-Exclusion looks like an n-cube)

+ ∅ $\binom{8}{4} = \frac{\cancel{8}^{2}\cdot 7\cdot \cancel{6}\cdot 5}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 70$

− A $\binom{3}{1}\binom{5}{3} = 3\cdot 10 = 30$

− B $\binom{3}{2}\binom{5}{2} = 3\cdot 10 = 30$

− C $\binom{5}{2}\binom{3}{2} = 30$

− D $\binom{5}{3}\binom{3}{1} = 30$

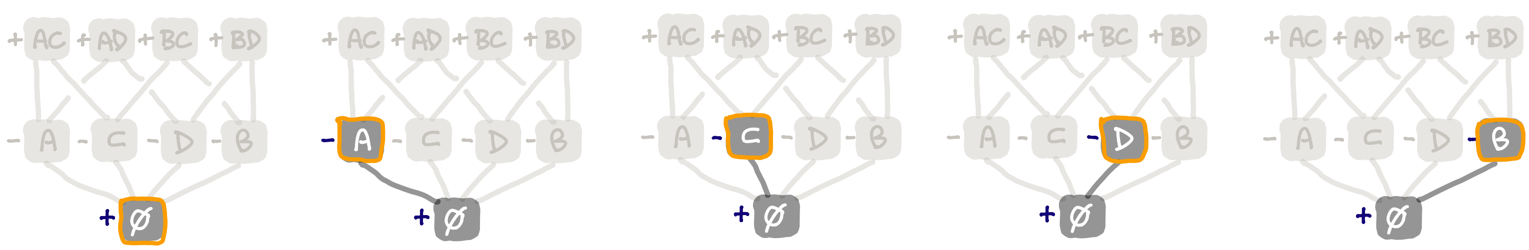
+ AC $\binom{3}{1}\binom{2}{1}\binom{3}{2} = 18$

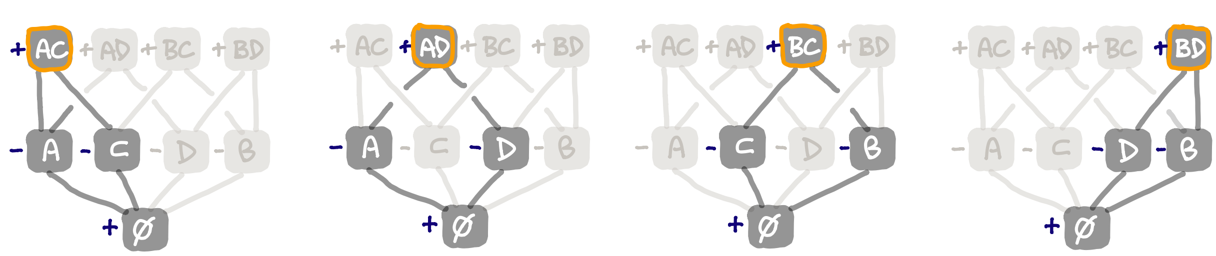
+ AD $\binom{3}{1}\binom{2}{2}\binom{3}{1} = 9$

+ BC $\binom{3}{2}\binom{2}{0}\binom{3}{2} = 9$

+ BD $\binom{3}{2}\binom{2}{1}\binom{3}{1} = 18$

$$\boxed{70 - 4\cdot 30 + 6\cdot 9 = 70 - 120 + 54 = 4}$$

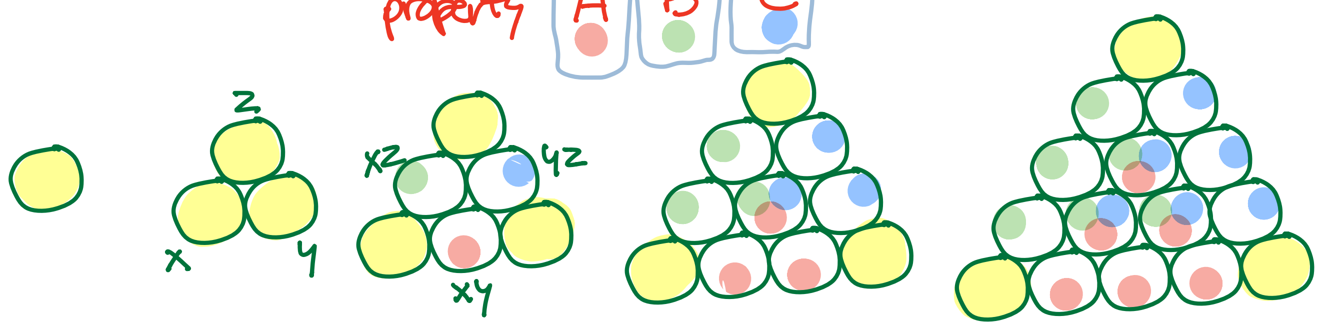




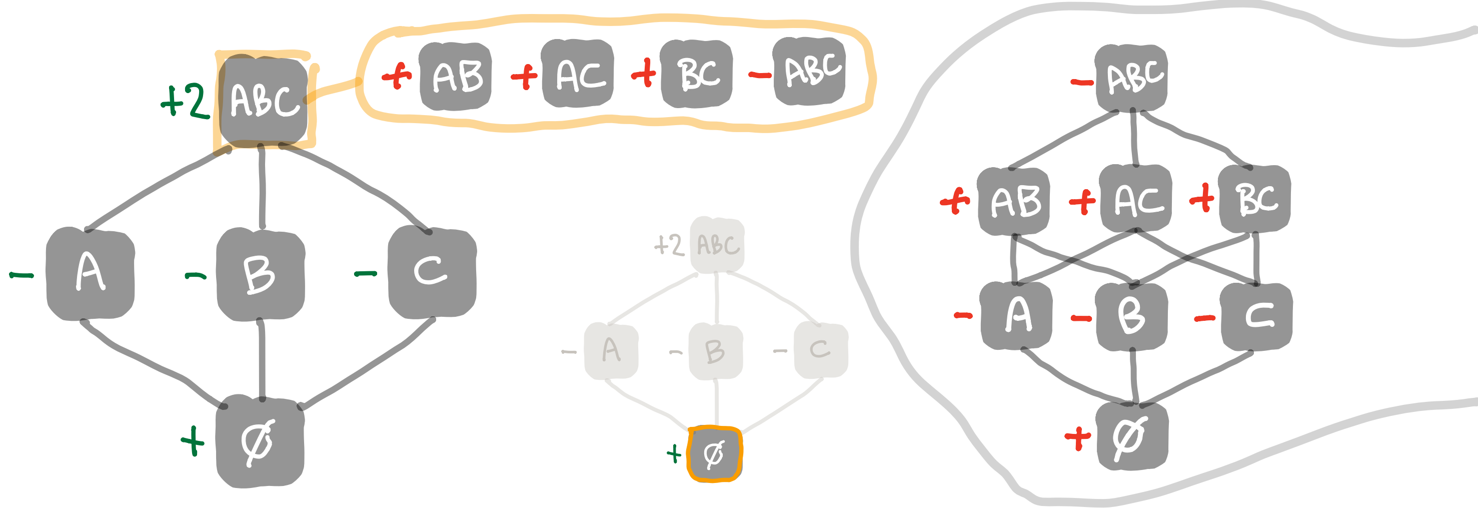
How many monomials of degree 4 in x, y, z are not divisible by any of xy, xz, yz?

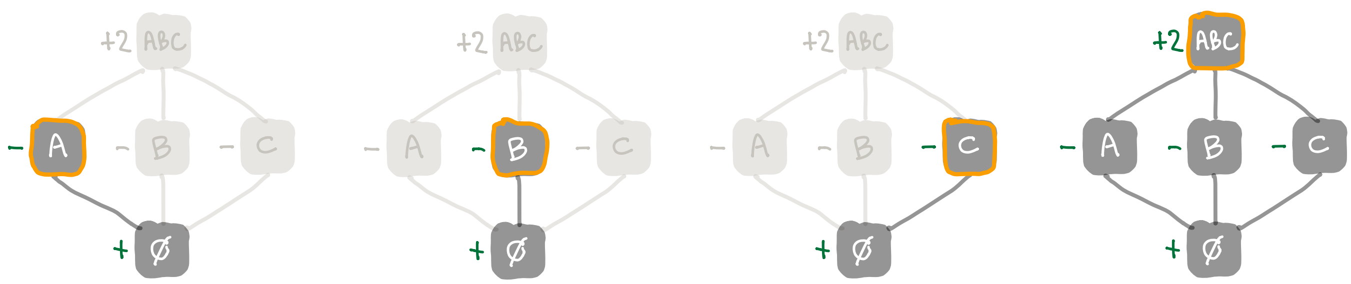
Soma cubes

	xy	xz	yz
property	A	B	C



$\emptyset$	$-A$	$-B$	$-C$	$+AB$	$+AC$	$+BC$	$-ABC$	
1	xy	xz	yz	xyz	xyz	xyz	xyz	
15	-6	-6	-6	$+3$	$+3$	$+3$	-3	$= \boxed{3}$



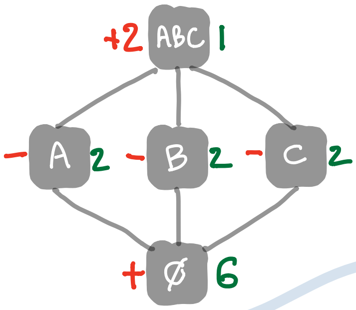


Can't have two of A, B, C without all three.
Where have we seen this before?

Permutations of 1, 2, 3

- A = 1 in 1st position
- B = 2 in 2nd position
- C = 3 in 3rd position

1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1



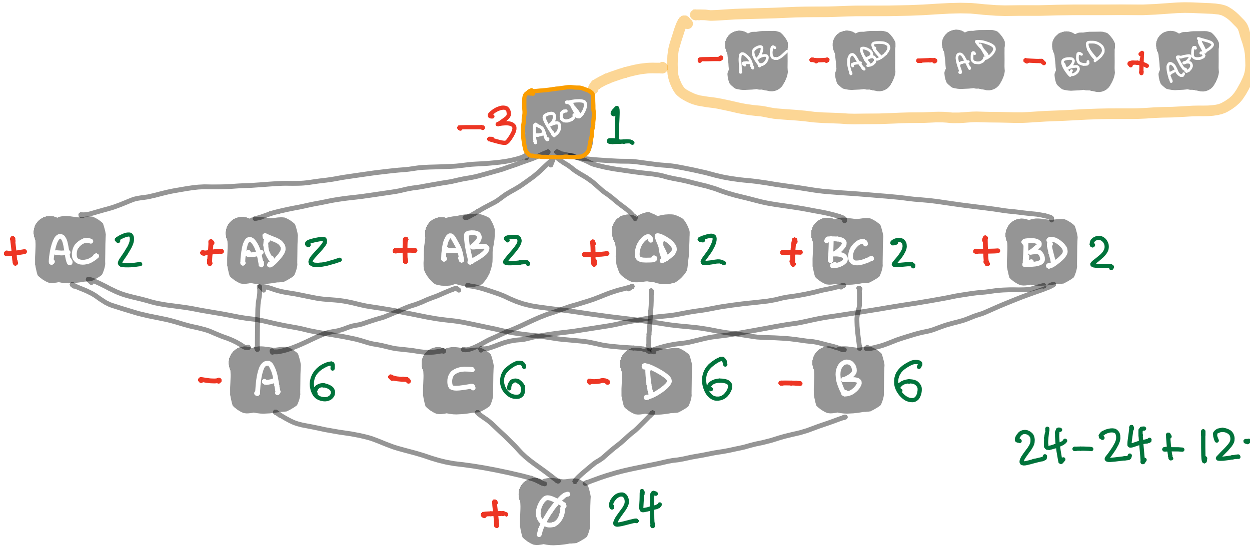
$\emptyset - A - B - C + 2ABC$

$6 - 2 - 2 - 2 + 2\cdot 1 = \boxed{2}$

A B C D

1 2 3 4	2 1 3 4	3 1 2 4	4 1 2 3
1 2 4 3	2 1 4 3	3 1 4 2	4 1 3 2
1 3 2 4	2 3 1 4	3 2 1 4	4 2 1 3
1 3 4 2	2 3 4 1	3 2 4 1	4 2 3 1
1 4 2 3	2 4 1 3	3 4 1 2	4 3 1 2
1 4 3 2	2 4 3 1	3 4 2 1	4 3 2 1

$\boxed{9}$



$24 - 24 + 12 - 3 = \boxed{9}$

How many permutations have no adjacent ascending pairs?

1	1 2	1 2 3	1 2 3 4	2 1 3 4	3 1 2 4	4 1 2 3
	2 1	1 3 2	1 2 4 3	2 1 4 3	3 1 4 2	4 1 3 2
		2 1 3	1 3 2 4	2 3 1 4	3 2 1 4	4 2 1 3
		2 3 1	1 3 4 2	2 3 4 1	3 2 4 1	4 2 3 1
		3 1 2	1 4 2 3	2 4 1 3	3 4 1 2	4 3 1 2
		3 2 1	1 4 3 2	2 4 3 1	3 4 2 1	4 3 2 1

$\emptyset - A - B - C + AB + AC + BC - ABC$

$24 - (6+6+6) + (2+2+2) - 1 = \boxed{11}$ ✓

$n! - \binom{n-1}{1}(n-1)! + \binom{n-1}{2}(n-2)! - \binom{n-1}{3}(n-3)! + \ldots$

$(n-1)!\left[n - (n-1) + \frac{n-2}{2} - \frac{n-3}{6} \ldots\right]$

$n=1 \qquad 0!\,[1] = 1$ ☑

$n=2 \qquad 1!\,[2-1] = 1$ ☑

$n=3 \qquad 2!\,[3-2+\frac{1}{2}] = 3$ ☑

$n=4 \qquad 3!\,[4-3+1-\frac{1}{6}] = 11$ ☑

How many permutations have no adjacent pairs, ascending or descending?

A B C

1 2 3 4

1	12	123	1234	2134	3124	4123
	21	132	1243	2143	3142	4132
		213	1324	2314	3214	4213
		231	1342	2341	3241	4231
		312	1423	2413	3412	4312
		321	1432	2431	3421	4321

$$(n-1)!\left[n-2\left((n-1)+\frac{n-2}{2}-\frac{n-3}{6}\ldots\right)\right]$$? a quick guess...

$n=1 \qquad 0!\,[1] = 1$ ☑

$n=2 \qquad 1!\,[2-2\cdot 1] = 0$ ☑

$n=3 \qquad 2!\,[3-2(2-\frac{1}{2})] = 0$ ☑

$n=4 \qquad 3!\,[4-2(3-1+\frac{1}{6})] = -2$?

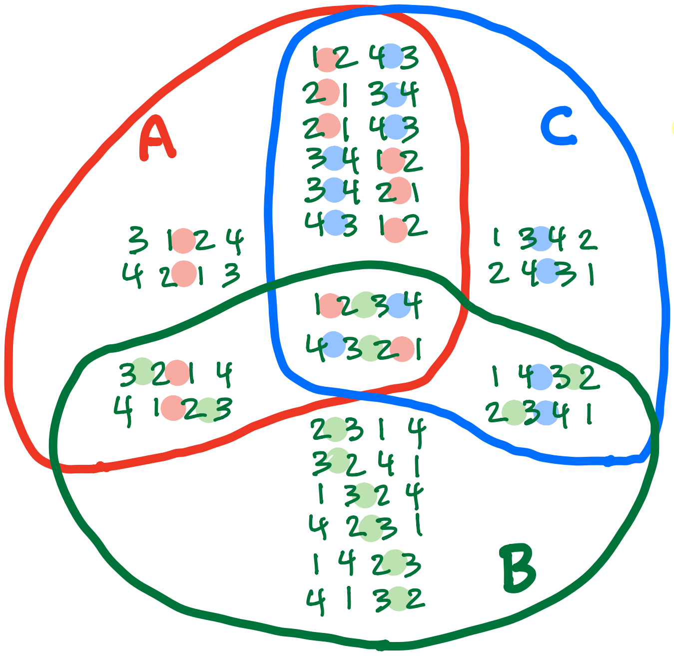
$\emptyset - A - B - C + AB + AC + BC - ABC$

$24-(12+12+12)+(4+4+4) \quad -2$

$24-36+12-2 = \boxed{-2}$?

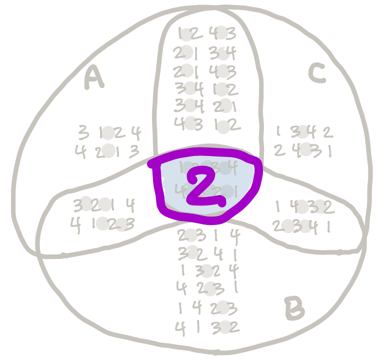
No. Too fast...

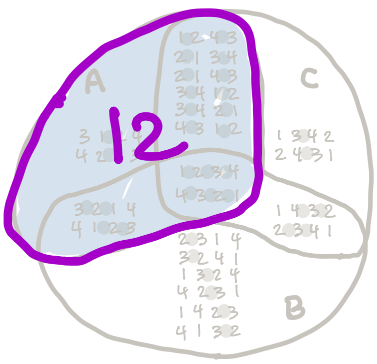
Check work.

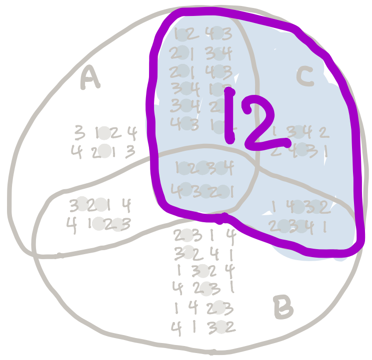


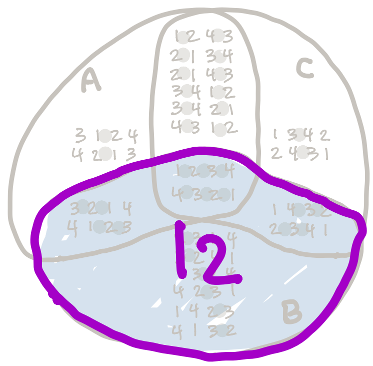
A B C
1 2 3 4

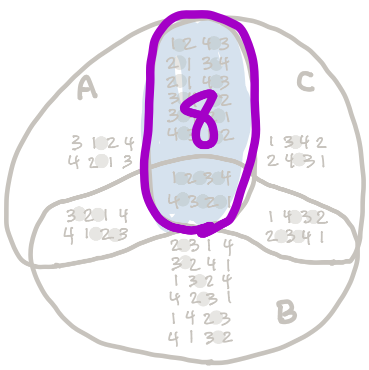
2413
3142

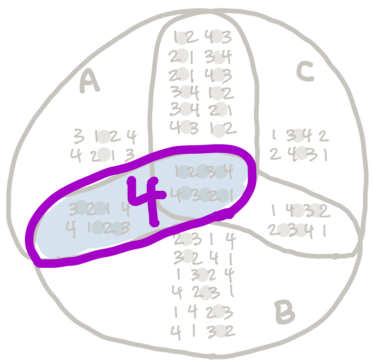


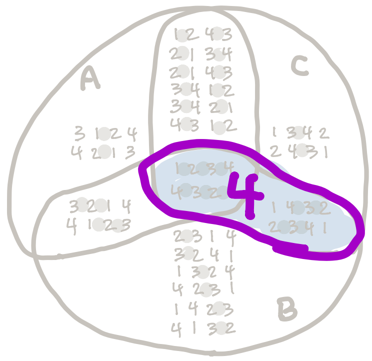












$$\emptyset - A - B - C + AB + AC + BC - ABC$$

$$24 - (12+12+12) + (4+8+4) \quad - 2$$

$$24 - 36 + 16 - 2 \;=\; \boxed{2} \checkmark$$