Let x, y degree 1
z degree 3

How many monomials of degree 12?

(HW1 Jan2014)

deg		0	1	2	3	4	5	6	7	8	9	10	11	12
1	x	1	1	1	1	1	1	1	1	1	1	1	1	1
1	y	1	2	3	4	5	6	7	8	9	10	11	12	13
3	z	1	2	3	5	7	9	12	15	18	22	26	30	35

z^4	1	1
z^3	4	x^3, x^2y, xy^2, y^3
z^2	7	$x^6, x^5y, x^4y^2, x^3y^3, x^2y^4, xy^5, y^6$
z	10	$x^9, x^8y, \dots xy^8, y^9$
1	13	$x^{12}, x^{11}y, x^{10}y^2, \dots, xy^{11}, y^{12}$

$$(1+x+x^2+\dots)(1+y+y^2+\dots)(1+z+z^2+\dots)$$

$\Downarrow$

$x=t \qquad y=t \qquad z=t^3$

$$(1+t+t^2+\dots)(1+t+t^2+\dots)(1+t^3+t^6+\dots)$$

$$\frac{1}{1-t} \qquad \frac{1}{1-t} \qquad \frac{1}{1-t^3}$$

$$\frac{1}{(1-t)^2(1-t^3)} = \boxed{\frac{1}{1-2t+t^2-t^3+2t^4-t^5}}$$

$$\begin{array}{r|l} & 1-2t+t^2 \\ \hline 1 & 1-2t+t^2 \\ -t^3 & -t^3+2t^4-t^5 \end{array}$$

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	5	7	9	12	15	18	22	26	30	35
							1	−2	1	−1	2	

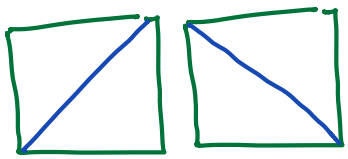
$$1-2t+t^2-t^3+2t^4-t^5=0$$
$$1=2t-t^2+t^3-2t^4+t^5$$
$$t^5-2t^4+t^3-t^2+2t=1$$

1 -2 1 -1 2

S 2014 Exam 1 [5]

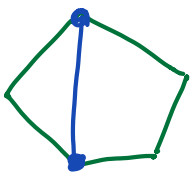
How many ways to cut an 8-gon into 3 pieces

$n=8$ $c=2$ cuts



$n=4$ $c=1$

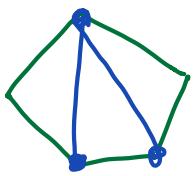
Catalan number $=2$



$n=5$
$c=1$

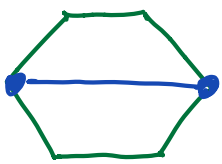
$\binom{5}{2}=5$

$\binom{n}{2}$



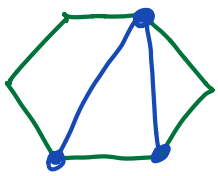
$c=2$

Catalan number $=5$



$n=6$
$c=1$

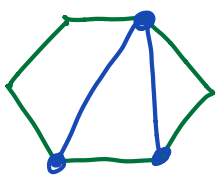
$\binom{6}{2}=15$



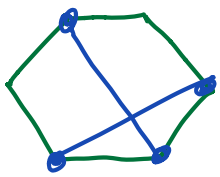
$n=6$
$c=2$

??

Use inclusion-exclusion

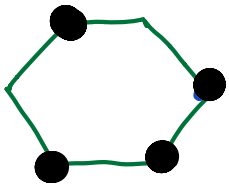


all
don't care about crossings

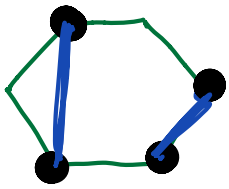


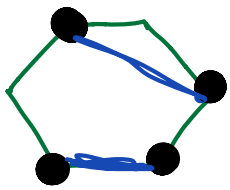
bad
they cross

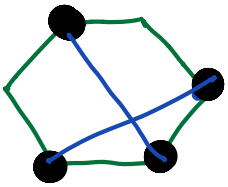
$$\binom{\binom{n}{2}-n}{2}-\binom{n}{4}$$

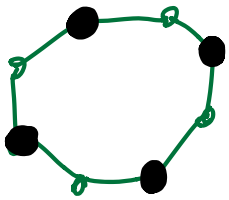


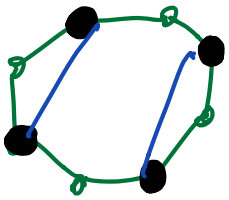
bad

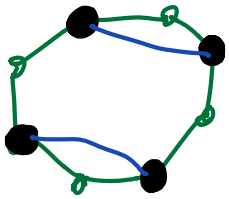


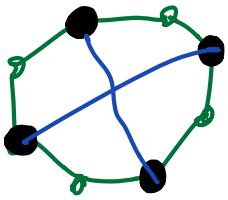












$$\binom{\binom{n}{2}-n}{2} - \binom{n}{4}$$

$n = 8$

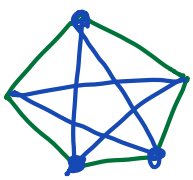
$\binom{8}{2} = 28 \quad \binom{8}{2} - 8 = 20$

$\binom{\binom{8}{2}-8}{2} = \binom{20}{2} = 190$

$\binom{8}{4} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70$

$190 - 70 = 120$

$n = 5$ expect 5



$\binom{5}{2} = 10$ edges

$\binom{5}{2} = 10$

$\binom{\binom{5}{2}-5}{2} = \binom{5}{2} = 10$

$\binom{5}{4} = 5 \quad 10 - 5 = 5$ ✓

$n = 6$

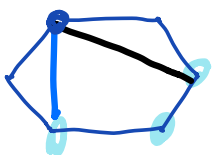
$\binom{6}{2} = 15$

$\binom{6}{2} - 6 = 9$

$\binom{\binom{6}{2}-6}{2} = \binom{9}{2} = 36$

$\binom{6}{4} = \binom{6}{2} = 15$

$36 - 15 = 21$

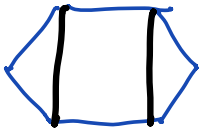


cuts meet

6 choices of vertex

× 3 choices pairs of opposite corners

18

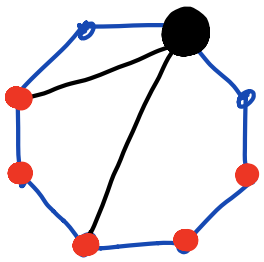


cuts don't meet
3 rotations

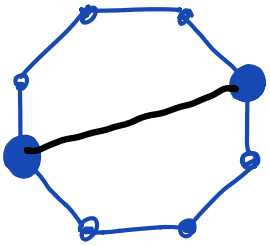
$18+3=21$ ✓

120

$n=8$



cuts meet
8 choices vertex ●
$\times \binom{5}{2}=10$ choices pair of opposite corners
80

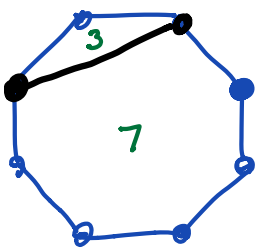


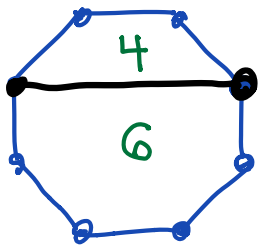
one diagonal
2 choose which half
$\times 5$ choose one cut in pentagon
10

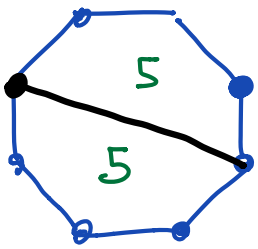
---

alt approach 1st cut, 2nd cut $\div 2$ for both orders

$\binom{8}{2}-8=20$







1st cut	8	+	8	+	4	= 20
2nd cut	$0+\binom{7}{2}-7$		$2+\binom{6}{2}-6$		$\binom{5}{2}-5)+(\binom{5}{2}-5)$	
	14		9		5 + 5	
			11		10	

112 + 88 + 40 $= 240/2 = 120$

✓

2015 HW1 [7]

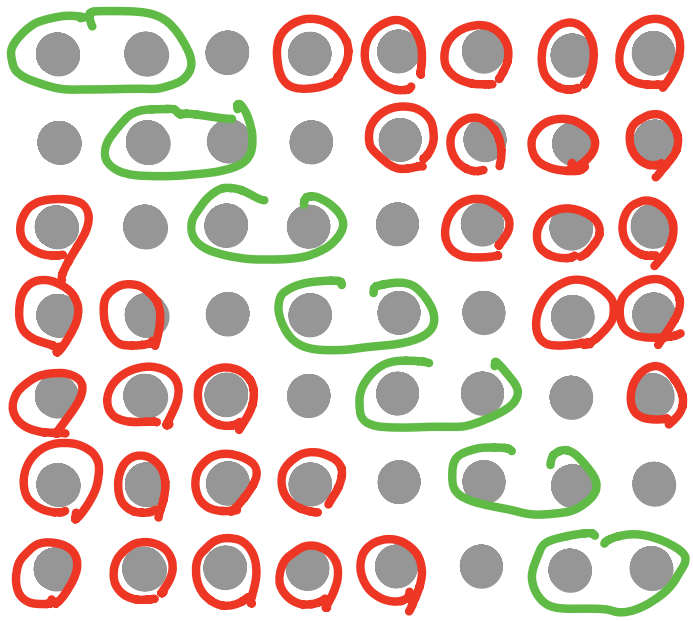
Choose 3 integers 1..8 none adjacent

$\binom{8}{3}$ in all, good and bad $\quad \binom{8}{3} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

— 23 7 2-1 clump

— 456 3 clump

23 7



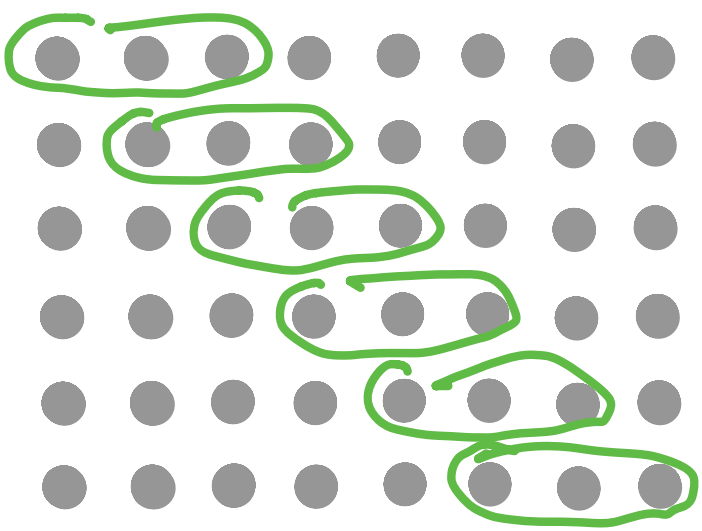
1: choose 2 clump

O =

$2 \cdot 5 + 5 \cdot 4 = 30$

$19 + 20$

2 clumps = 30



3-clumps 6

$56 - 30 - 6 = 20$