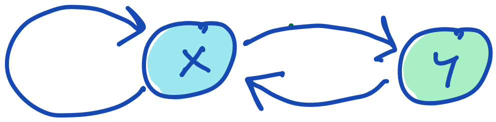
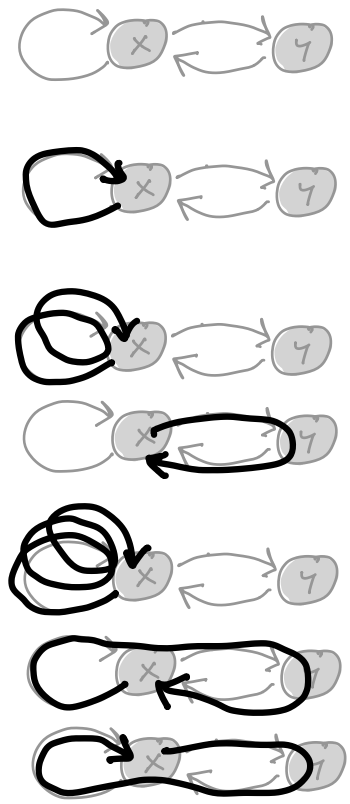


paths x to itself
length n



n=0 1

n=1 1

n=2 2

n=3 3

want numbers

n	0	1	2	3	...
$f(n)$	1	1	2	3	...

$g(t) = 1 + 1t + 2t^2 + 3t^3 \dots$

$$f(n) = \begin{cases} 0, & n<0 \quad ✓* \\ 1, & n=0 \quad ✓* \\ f(n-1)+f(n-2), & n>0 \end{cases}$$

How does $g(t)$ see this pattern?

	n	-2	-1	0	1	2	3	...
$f(n-1)$	$tg(t)$			$t($	1	$+ \square t$	$+ \square t^2 + \square t^3 + \dots)$	
$+ f(n-2)$	$t^2g(t)$			$t^2($		1	$+ \square t + \square t^2 + \square t^3 + \dots)$	
$f(n)$ *	$g(t)$			1	$+ \square t$	$+ \square t^2$	$+ \square t^3 + \dots$	

t column t^2 column

$$g(t) = \sum_{n=0}^{\infty} f(n) t^n$$

$f(n) = f(n-1) + f(n-2)$ *

$$\Rightarrow \boxed{g(t) = tg(t) + t^2 g(t) + 1}$$

checklist:

$n<0$: no negative powers of t on either side

$n=0$: t^0 $g(t)$ on left $= 1$ on right $\Rightarrow$ $g(t)$ starts with 1

$n>0$: t^1

t^2

$$g(t) - t\,g(t) - t^2 g(t) = 1$$

$$(1-t-t^2)\,g(t) = 1$$

$$g(t) = \frac{1}{1-t-t^2} = \boxed{\frac{1}{1-(t+t^2)}}$$

what does this mean?

$$1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x} \quad \text{(geometric series)}$$

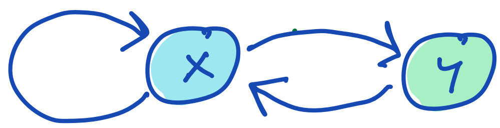
$$1 + a + a^2 + a^3 + \ldots = \frac{1}{1-a}$$

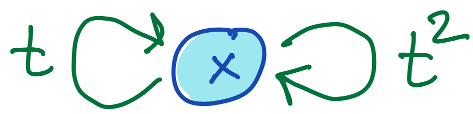
$$\boxed{1 + (t+t^2) + (t+t^2)^2 + (t+t^2)^3 + \ldots = \frac{1}{1-(t+t^2)}}$$

think what this means

$1 \quad t \quad t^2 \quad t^2 \quad 2t^3 \quad t^4 \quad t^3 \quad 3t^4 \quad 3t^5 \quad t^6 \quad \ldots$

$1 \quad t \quad 2t^2 \quad 3t^3$





$g(t)$ collects all possible paths, tagged by t^n

do t or t^2 zero or more times

exactly what this says:

$$\boxed{1 + (t+t^2) + (t+t^2)^2 + (t+t^2)^3 + \ldots = \frac{1}{1-(t+t^2)}}$$