March 9, 2021 Counting with symmetries on polytopes.

Symmetries of space

Linear Algebra

w/o angle, length
then add these <fig> f.g

xkcd.com

chirality

orthonormal basis $\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}$

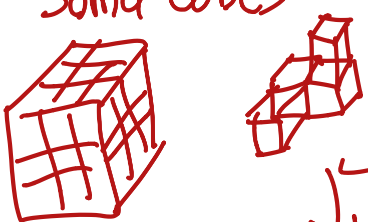
$v_1 \perp v_2$ $|v_i| = 1$
$v_1 \perp v_3$ $v_i \cdot v_i = 1$
$v_2 \perp v_3$

$v_i \perp v_j = v_i \cdot v_j = 0$

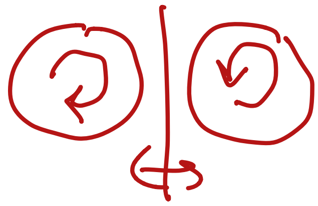
$$\underbrace{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}}_{A^T} \underbrace{\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}}_{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^{-1} = A^T$ $\boxed{\det(A) = 1}$

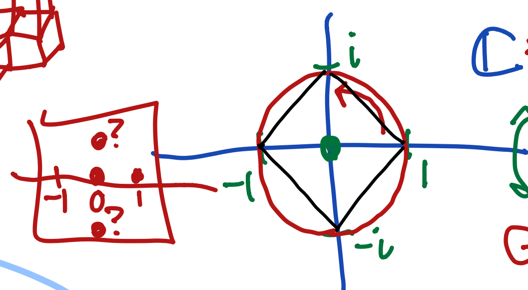
Soma cubes



rotations in space



$\mathbb{C} = \mathbb{R}^2$



$\overline{a+bi} = a-bi$
$\overline{i} = -i$

$G = \{ c \in \mathbb{C} \mid |c| = 1 \} = SO(2)$
c·d rotations

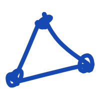
$x^2 + 1 = 0$
$(x+i)(x-i) = 0$

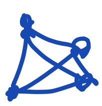
$O(n)$ = orthonormal $\mathbb{R}^n \to \mathbb{R}^n$ matrices

$SO(n) =$ " det = 1
rotations

n-simplex

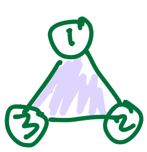
interval  1-simplex

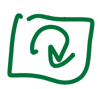
triangle  2-simplex

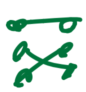
tetrahedron  3-simplex

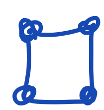
symmetric group : permutations of $\{1, \ldots, n\}$ S_n all

A_n even



geometric view 

permutation view 

$|S_4| = 24 = 4!$

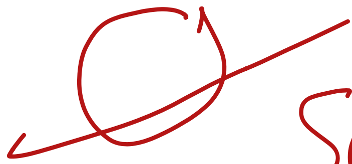
$|G| = 8$

$|S_4| = 24$
$|A_4| = 12$

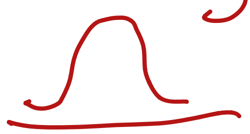
S_5
"A_5 can't be factored" A_n $n \geq 5$

$ax^2+bx+c=0 \Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $\quad \sqrt{}$

$\overline{a+bi}$
$=a-bi$



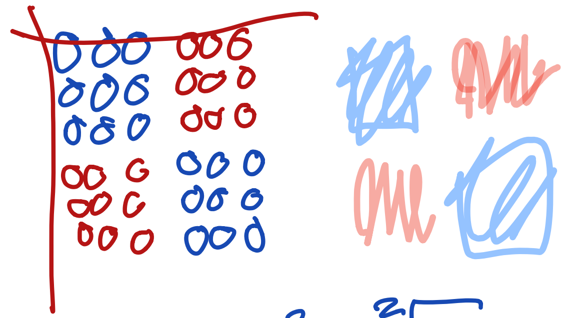
$\int e^{x^2}dx$



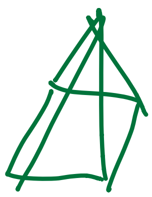
erf(x)

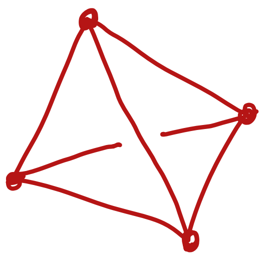
$\sqrt{}$ $\sqrt[3]{}$

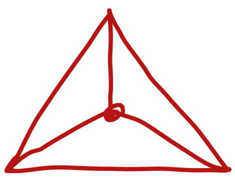
$\sqrt[4]{}$

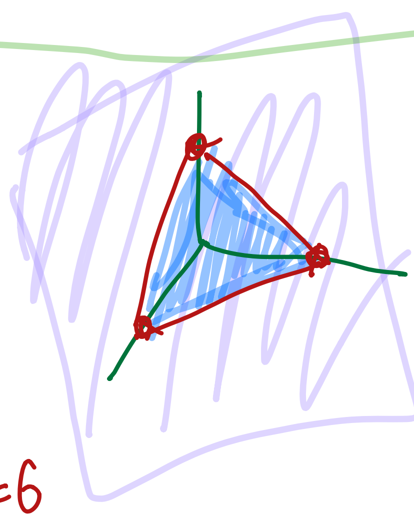


2 $\sqrt[2]{}$







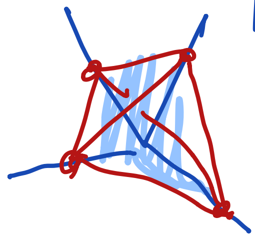


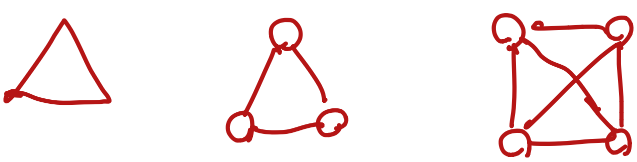
all $v=(x,y,z)$

$x,y,z\geq 0$

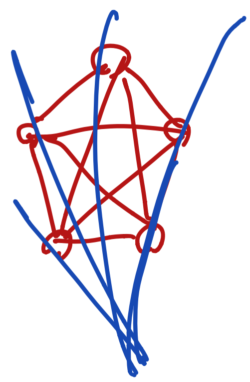
$x+y+z=1$

$\mathbb{R}^4$

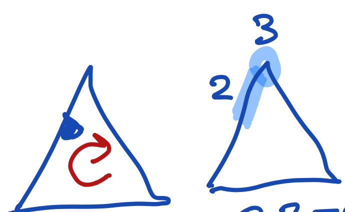




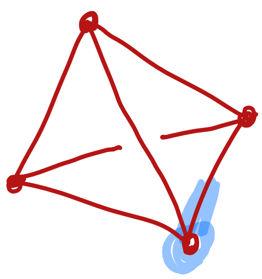
$\binom{4}{2}=6$

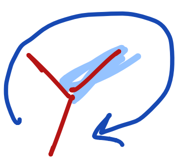


$\binom{5}{2}=10$



$3\cdot 2=6$





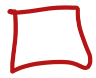
mark it to destroy symmetry

count choices

4 choices of corner

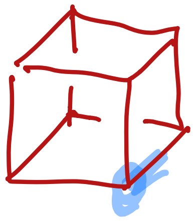
$\times$ 3 choices of edge meeting that corner

12



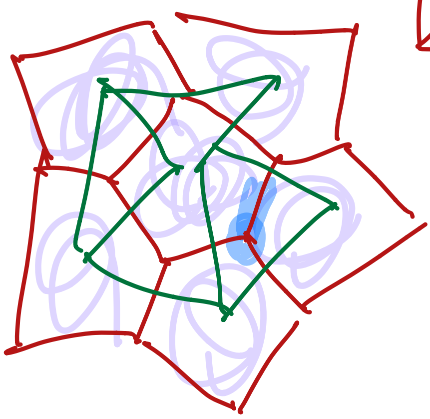
$|G| = 8$

$|S_4| = 24$



$|G| = 8 \cdot 3 = 24$

$|S_8| = 8!$



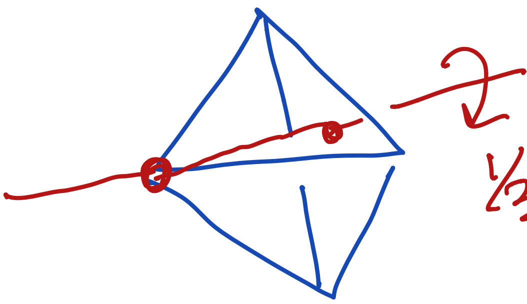
$12 \cdot 5/3 = 20$

$|G| = 20 \cdot 3 = 6$

$G = A_5$

\#ways k-color faces of a tetrahedron up to symmetry

$|G| = 12$



1/3 turn

1 do nothing, identity

8 1/3 turns — 4 vertices ×2 turns

3 1/2 turns

$\overline{12}$