Combinatorics
January 18, 2022

Class #1

binomial coefficients and inclusion-exclusion

$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$ integer

$0! = 1$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

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permutations $n=3$ $\{1,2,3\}$

123
132
213
231
312
321

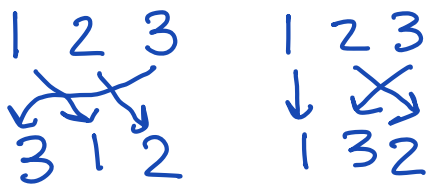
$n!$

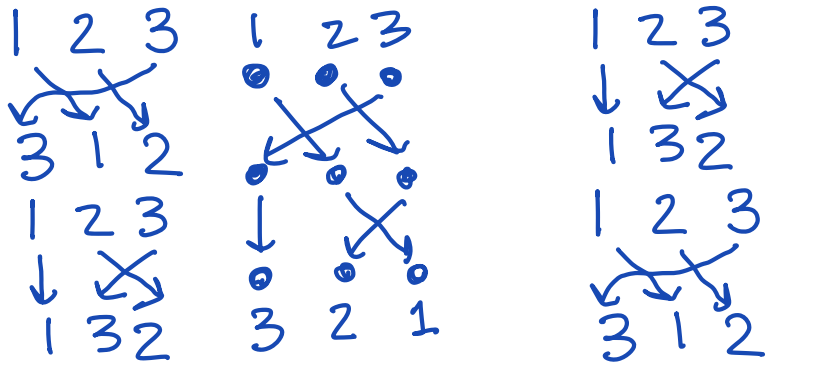
$3! = 3\cdot 2\cdot 1 = 6$

linear algebra

$$\begin{matrix} & 1 & 2 & 3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} & 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix} \begin{matrix} & 1 & 3 & 2 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix} = \begin{matrix} & 3 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{matrix}$$



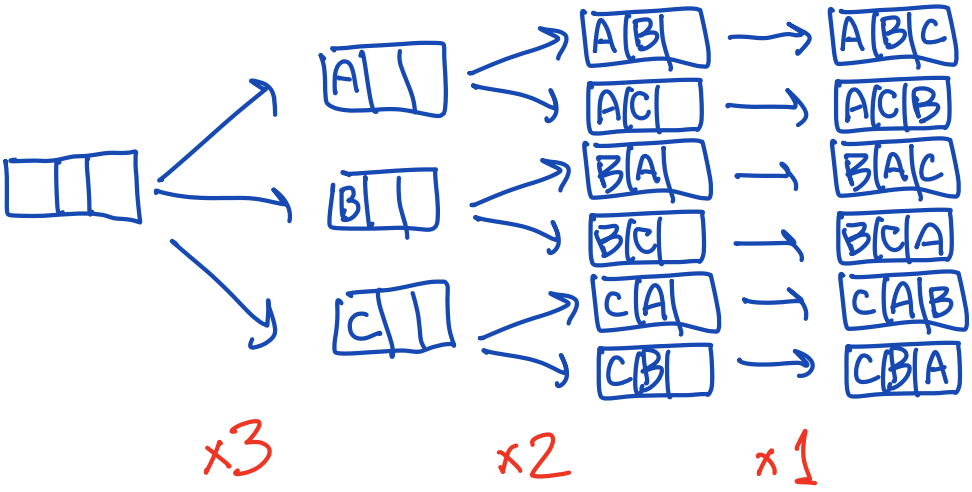


$n=3$

A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A

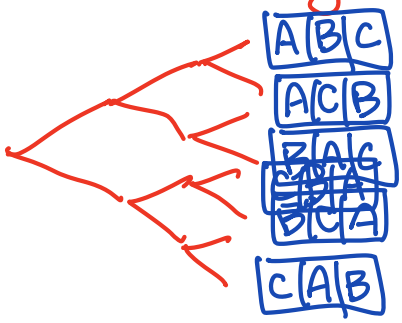
(23)

$n!$



3·2·1

n log n steps to sort n elements

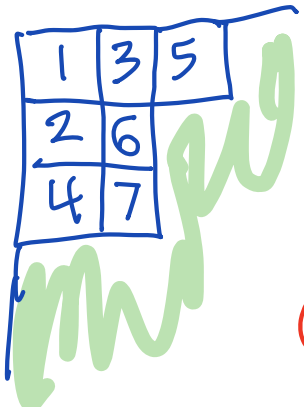


n choose k $\binom{n}{k}$

= # subsets size k from n things

$\binom{4}{2}$

	A	B	C	D	
	X	X			AB
	X		X		AC
	X			X	AD
		X	X		BC
		X		X	BD
			X	X	CD



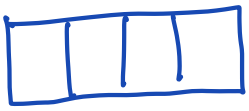
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$= \frac{n_k}{k!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$

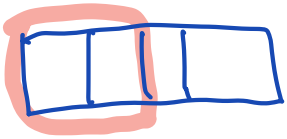
$$\binom{4}{2} = \frac{4!}{2!\,2!} \quad \frac{4\cdot 3\cdot 2\cdot 1}{2\cdot 1\,\cdot\,2\cdot 1} = \frac{4\cdot 3}{2\cdot 1} = \frac{12}{2} = 6$$

counting technique: overcount, divide by overcount

ABCD $\binom{4}{2}$



write perm of ABCD (4) $4! = 24$

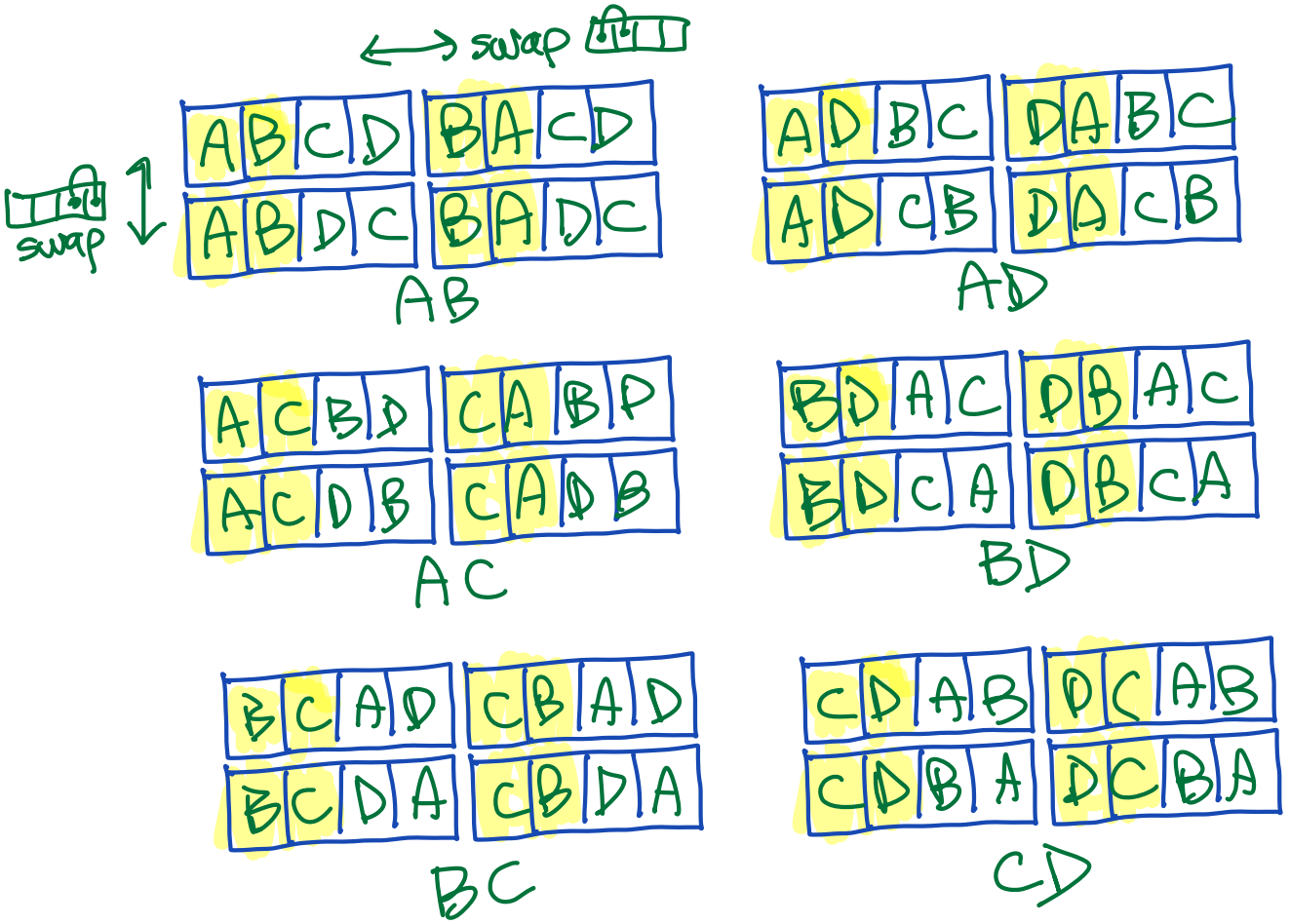


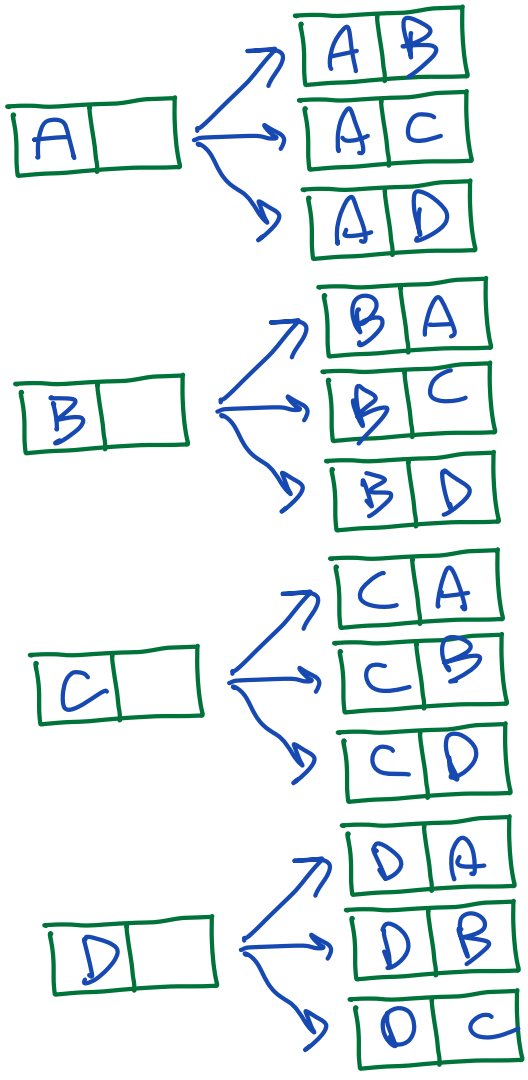
choose first two entries as a <u>set</u>

ignore order

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

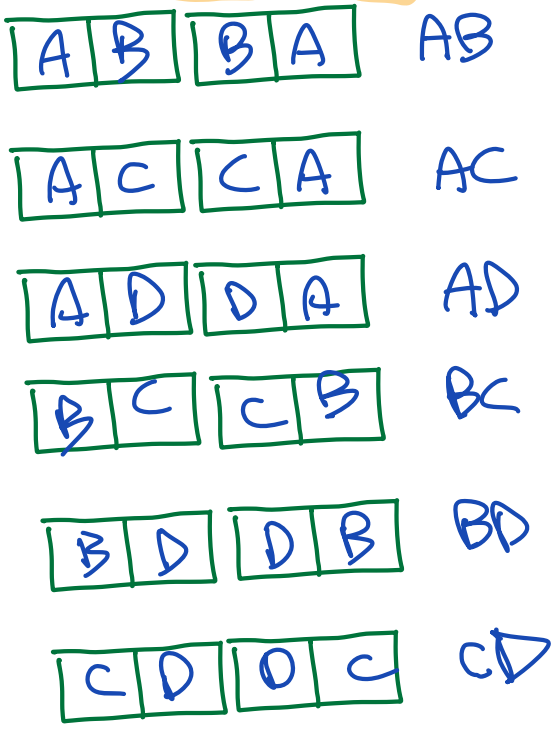
ABCD

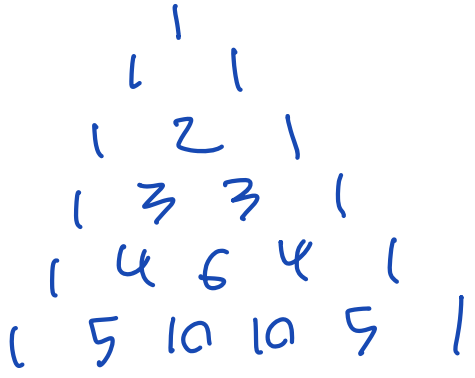


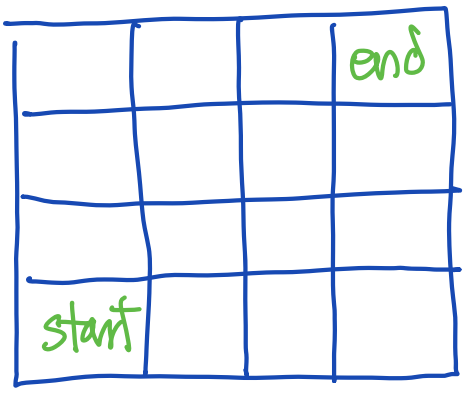


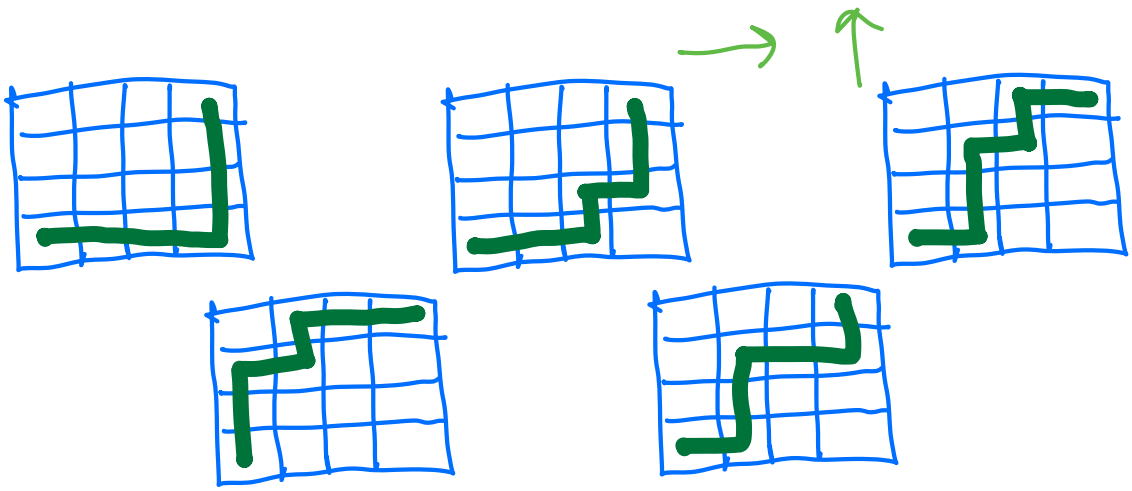
fill in two from ABCD

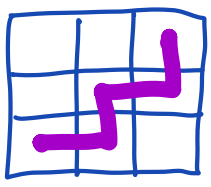
$$\binom{n}{k} = \frac{n_k}{k!} = \frac{n(n-1)\cdots(n-k+1)}{k!}$$





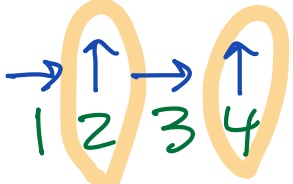




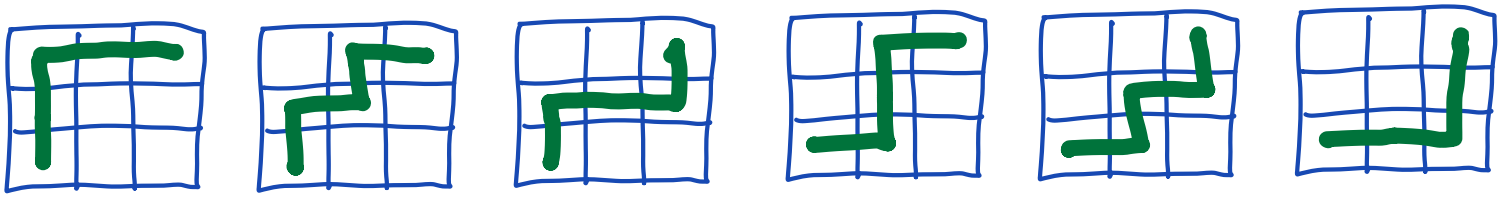


24

→ → ↑ ↑



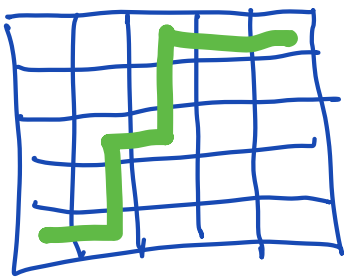
which steps are ↑ (not →)



12 13 14 23 24 34

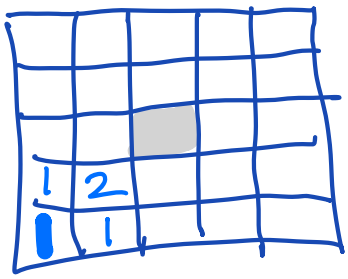
inclusion-exclusion

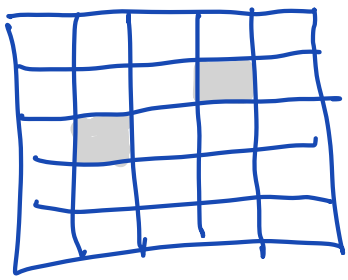
How many paths if certain squares are forbidden?



→4 ↑4

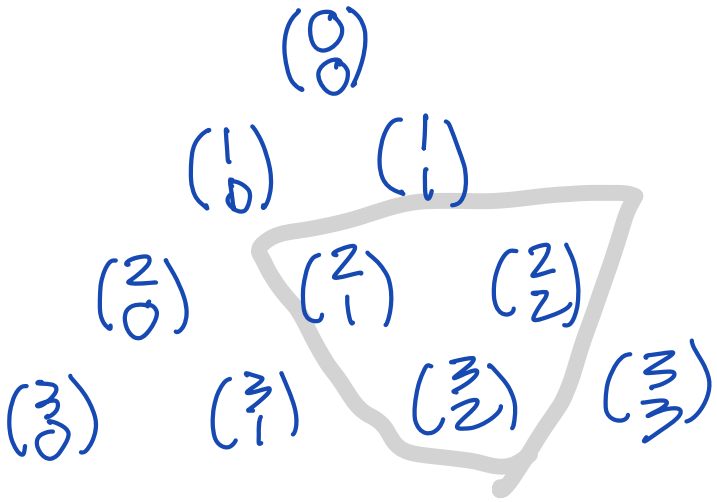
$$\binom{8}{4} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70$$





1	5	15	35	70
1	4	10	20	35
1	3	6	10	15
1	2	3	4	5
1	1	1	1	1

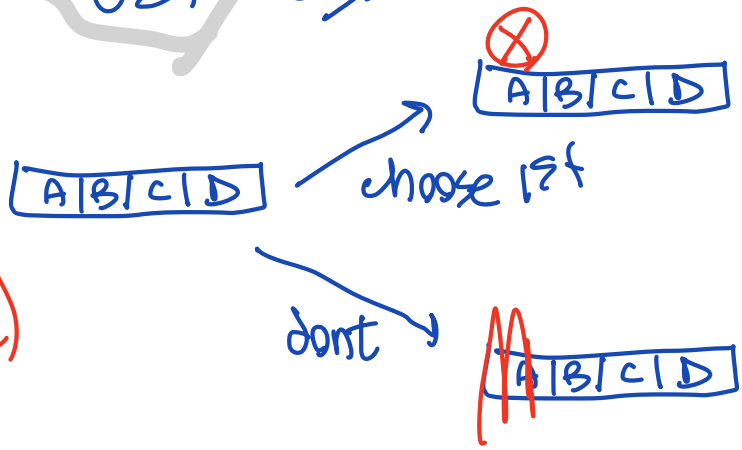
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1



$$\binom{3}{2} = \binom{2}{1} + \binom{2}{2}$$

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

choose 1 | don't



$\binom{4}{2}$