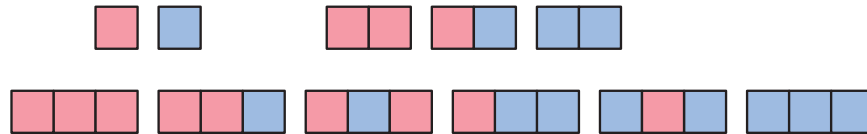# Exam 2

Combinatorics, Dave Bayer, April 6-10, 2022

Please show all of your work. You will be graded for both your answers and your explanations. You need not complete the entire exam; the questions vary in difficulty.

[1] How many ways can we color the cells of a strip of n squares using at most k colors, counting two patterns as the same if one is a reversal of the other?


[2] How many ways can we color the cells of this beehive using at most k colors, up to the dihedral group of rotations and flips? Confirm your answer for $k = 2$, by finding all patterns up to symmetry.
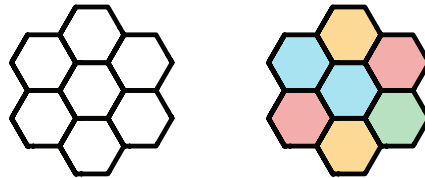
[3] How many ways can we color the edges of a cube using at most k colors, up to the group of rotational symmetries? Can you check your answer for $k = 2$?
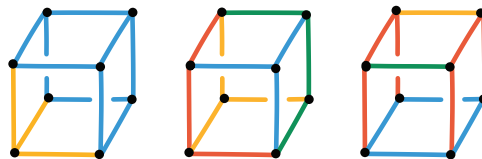
[4] Let $f(n)$ be the number of ways of dissecting an n-gon by at least one cut, up to the dihedral group of rotations and flips. As shown, $f(4) = 1$ and $f(5) = 2$. Find $f(6)$ two ways, by drawing the cases by hand and by using Burnside's lemma.
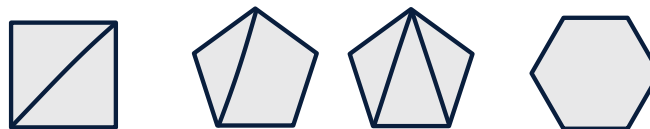
[5] How many ways can we color the faces of a cube using at most k colors, up to the group of symmetries generated by rotations and reflections ("look in the mirror")?