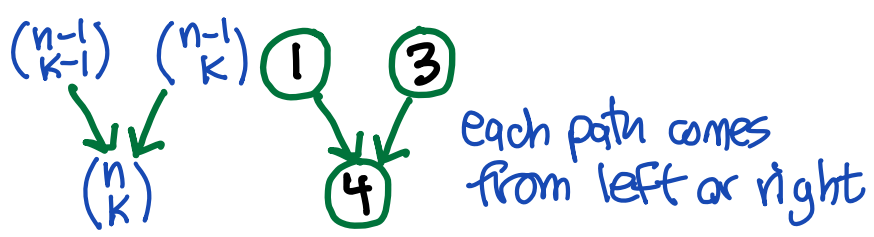
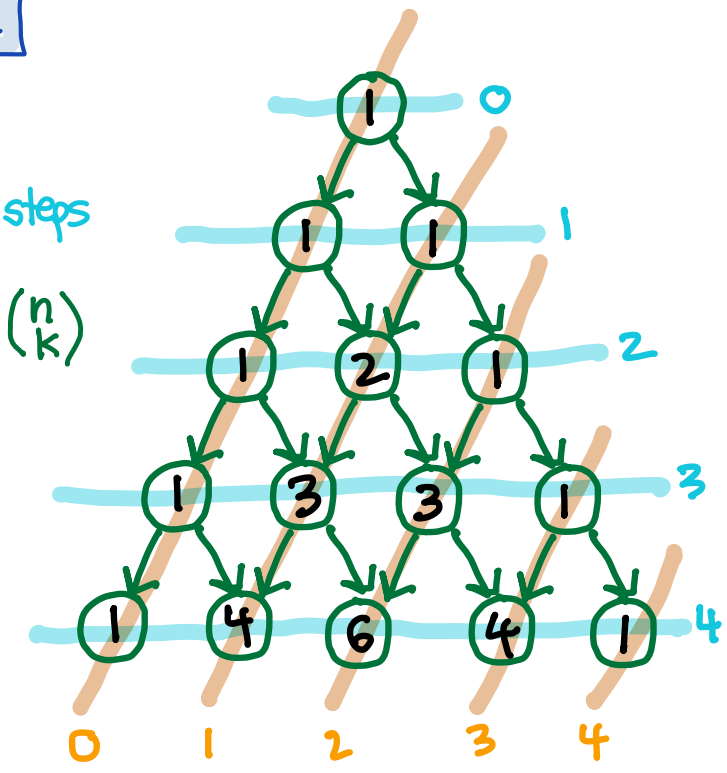
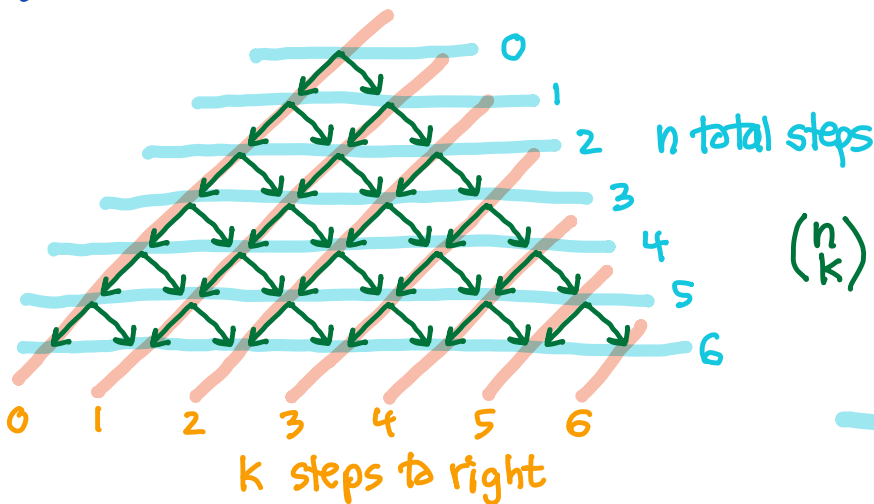


Combinatorics #3, January 25, 2022

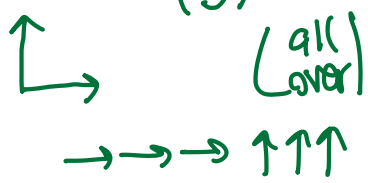


$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

1	4	10	20
1	3	6	10
1	2	3	4
1	1	1	1

start

$$\binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$$



inclusion-exclusion

1	5	9	17	34
1	4	4	8	17
1	3	0	4	9
1	2	3	4	5
1	1	1	1	1

avoid

=

1	5	15	35	70
1	4	10	20	35
1	3	6	10	15
1	2	3	4	5
1	1	1	1	1

free-for-all

-

		6	18	36
		6	12	18
1	3	6	6	6
1	2	3		
1	1	1		

must use

34

$$\binom{8}{4} - \frac{28 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70$$

$$\binom{4}{2} \binom{4}{2} = 6 \cdot 6$$

1	1	2	4	7
1		1	2	3
1	1	1	1	1

avoid

1	3	6	10	15
1	2	3	4	5
1	1	1	1	1

all

	2	4	6	8
1	2	2	2	2
1	1			

use

1	2	3	4
1	1	1	1

1	2
1	1

$$\binom{6}{2} - \binom{2}{1}\binom{4}{1}$$

$$\frac{6 \cdot 5}{2!} - 2 \cdot 4$$

$$7 = 15 - 8$$

1	2	4	4	7
1	1	2	B	3
1	A	1	2	3
1	1	1	1	1

$\emptyset$  = no properties or more

A = at least property A

B = at least property B

AB = at least properties A and B

$$\text{avoid} = \emptyset - A - B + AB$$

$$7 = 35 - 20 - 20 + 12$$

1	4	10	20	35
1	3	6	10	15
1	2	3	4	5
1	1	1	1	1

$\emptyset$

	2	6	12	20
	2	4	6	8
1	2	2	2	2
1	1			

A

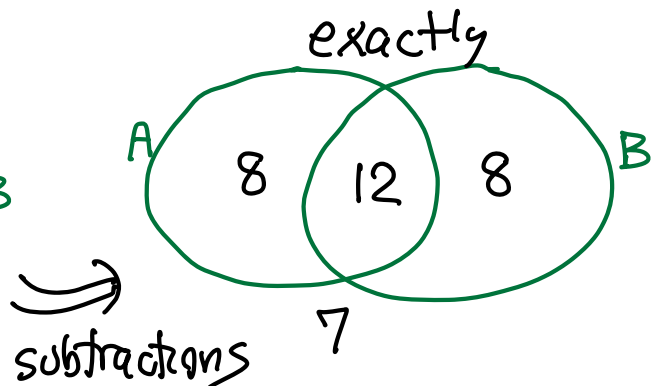
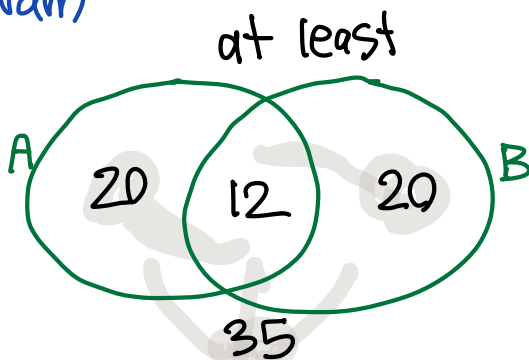
			10	20
1	3	6	10	10
1	2	3	4	
1	1	1	1	

B

			6	12
	2	4	6	6
1	2	2	2	
1	1			

AB

Venn diagram





1..30

$$\emptyset - A - B - C + AB + AC + BC - ABC$$

$\frac{30}{1}$	$\frac{30}{2}$	$\frac{30}{3}$	$\frac{30}{5}$	$\frac{30}{6}$	$\frac{30}{10}$	$\frac{30}{15}$	$\frac{30}{30}$
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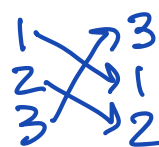
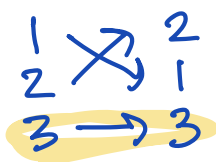
all multiples of

$$8 = 30 - 15 - 10 - 6 + 5 + 3 + 2 - 1$$

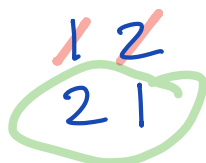
$-31$

Hat check problem

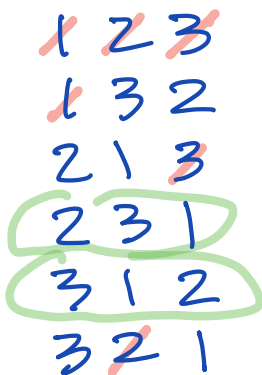
How many permutations have no fixed points?



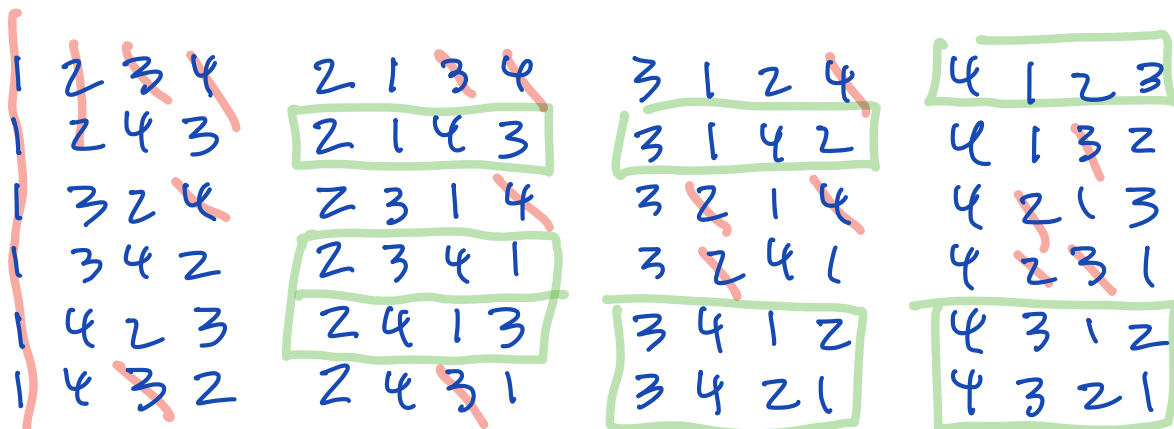
n=2



n=3



n=4



Find formula for  $n$  agrees with

$n$	2	3	4
$\#$	1	2	9
	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{9}{24}$