Combinatorics March 1, 2022

Burnside's lemma

$$|X/G| = \frac{1}{|G|}\sum_{g\in G}|X^g|$$

$$\sum_{g\in G}|X^g| = |\{(g,x) \mid gx=x\}| = \sum_{x\in X}|G_x|$$

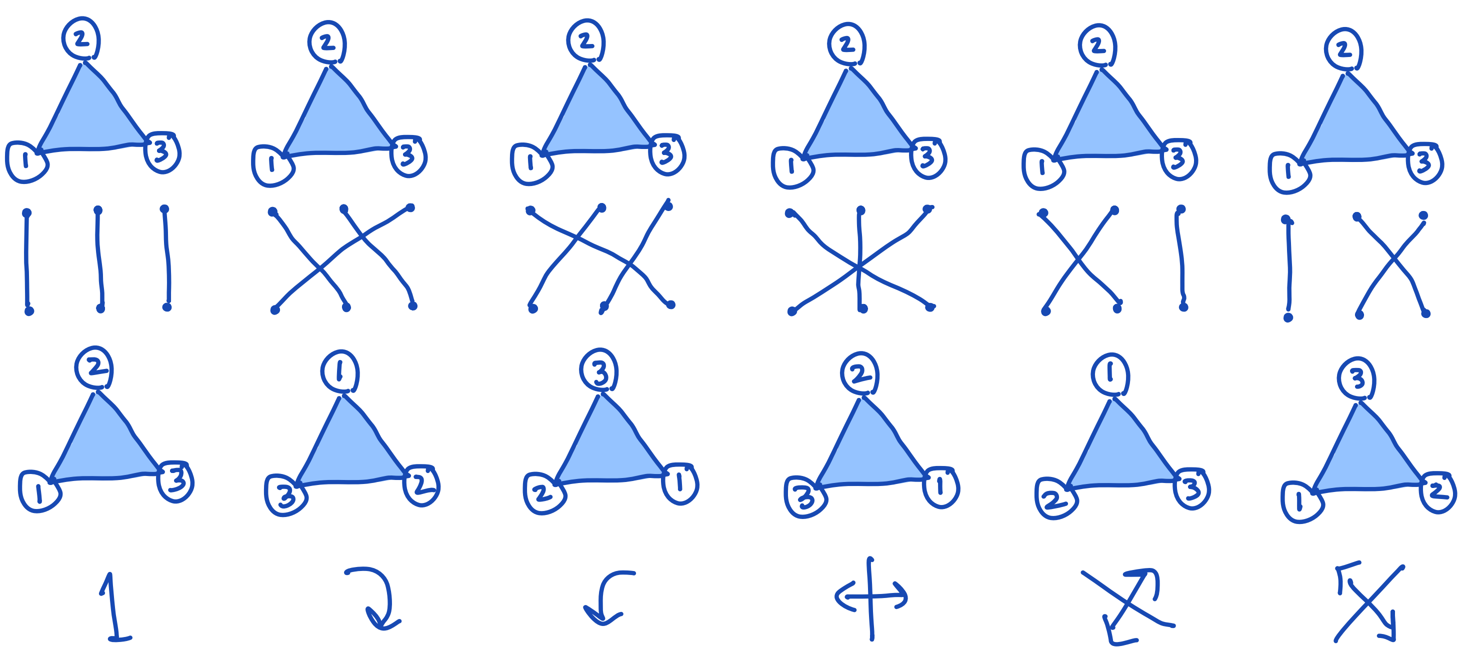
$$= \sum_{x\in X}\frac{|G|}{|Gx|} = |G|\sum_{x\in X}\frac{1}{|Gx|} = \sum_{A\in X/G} 1 = |X/G|$$

Or, every orbit of G acting on X has $|G|$ fixed points

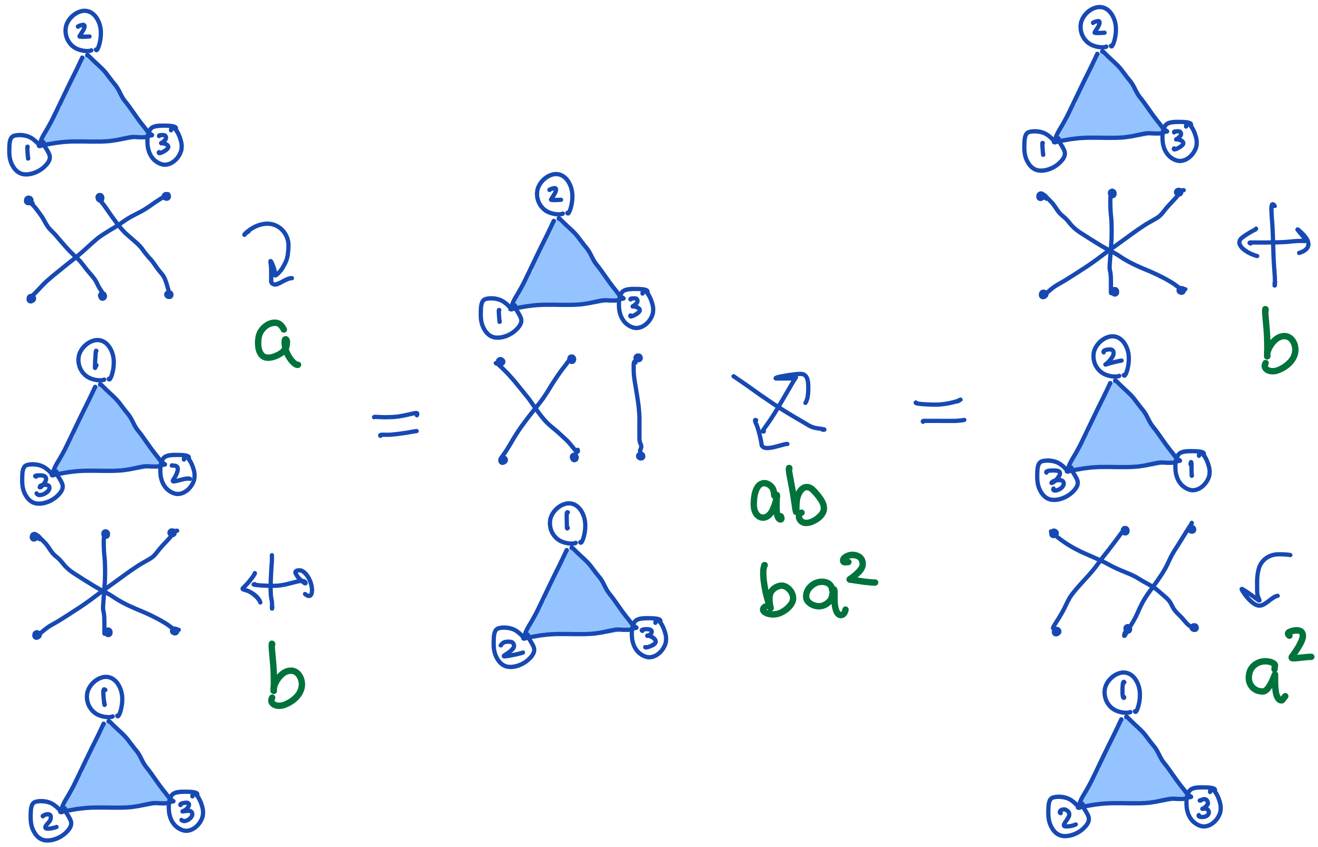
$$|X/G| = \#\text{orbits} = \frac{1}{|G|}(\text{total \# fixed points})$$

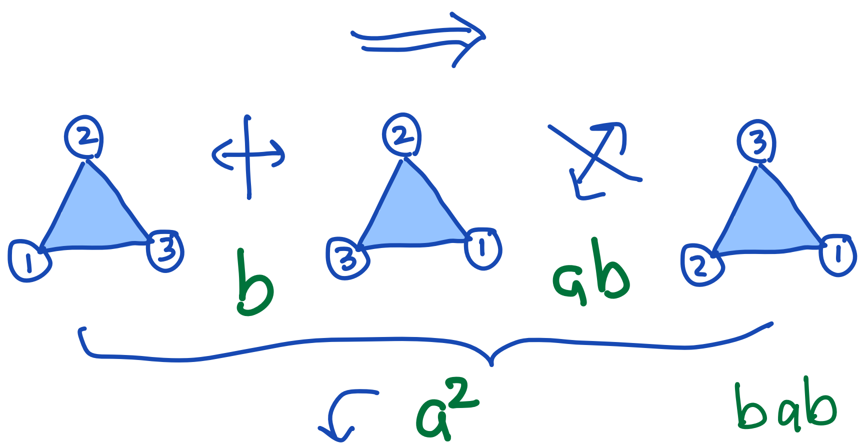
$G = S_3$ permutations of $\{1,2,3\}$

symmetries of Δ (rotations, flips)



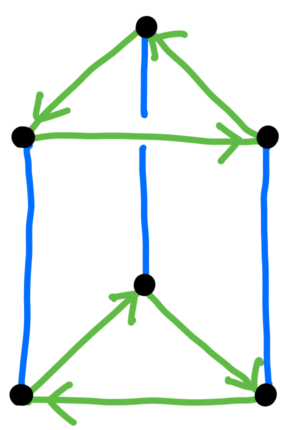
1	a	a^2	b	ab	a^2b





$bab = bba^2 = a^2$

$S_3 = \langle a, b \mid a^3 = b^2 = 1,\ ab = ba^2 \rangle$



Cayley diagram

$\longrightarrow$ a

— b