Combinatorics prep, 3/10/22

How many ways to place k markers on an n×n board up to rotations, flips?

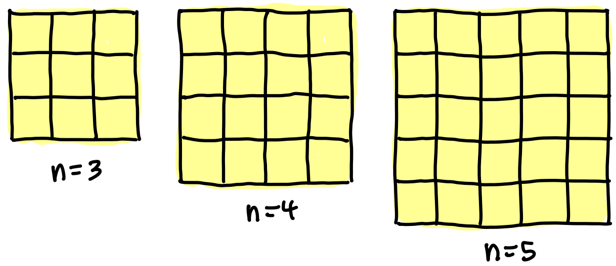
$D_4 = \{ 1, \curvearrowright, \underbrace{\circlearrowright, \circlearrowleft}_{\text{same}}, \underbrace{\leftrightarrow, \updownarrow}_{\text{same}}, \underbrace{\nearrow\!\!\!\swarrow, \nwarrow\!\!\!\searrow}_{\text{same}} \}$ $\qquad |M| = \frac{1}{|G|} \sum_{g \in G} |X^g|$

= choose 1

= choose 2

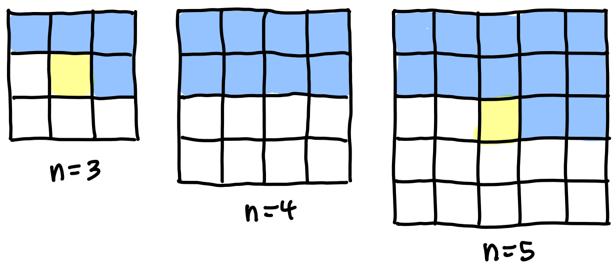
= choose 4

g=1



$\binom{n^2}{k}$

g= ↻



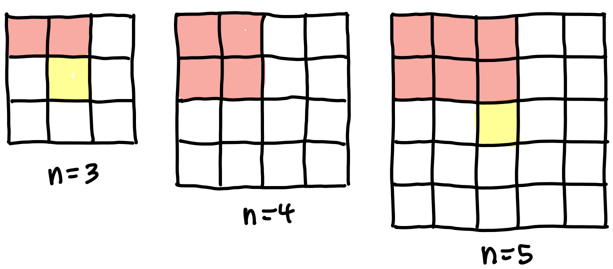
$m = \lfloor \frac{n^2}{2} \rfloor$ = #

k=2 $\quad m$

k=3 $\quad$ [n odd?] m

k=4 $\quad \binom{m}{2}$

g= ↺



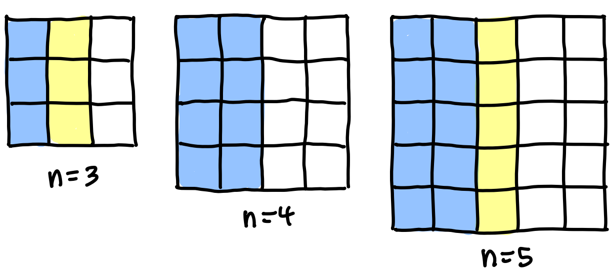
$m = \lfloor \frac{n^2}{4} \rfloor$ = #

k=2 $\quad 0$

k=3 $\quad 0$

k=4 $\quad m$

g= ↔



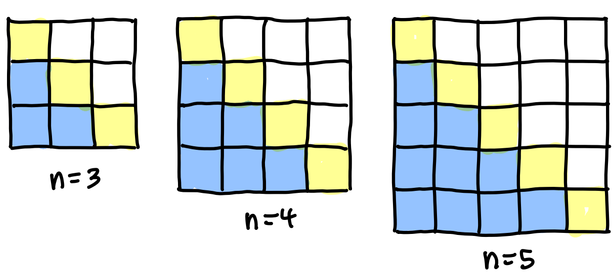
$m = n \lfloor \frac{n}{2} \rfloor$ = #

k=2 $\quad m +$ [n odd?] $\binom{n}{2}$

k=3 $\quad$ [n odd?] mn

k=4 $\quad \binom{m}{2} +$ [n odd?] $\left(m\binom{n}{2} + \binom{n}{4} \right)$

g= ⤢



$m = \frac{n^2 - n}{2}$ = #

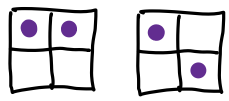
k=2 $\quad m + \binom{n}{2}$

k=3 $\quad mn$

k=4 $\quad \binom{m}{2} + m\binom{n}{2} + \binom{n}{4}$

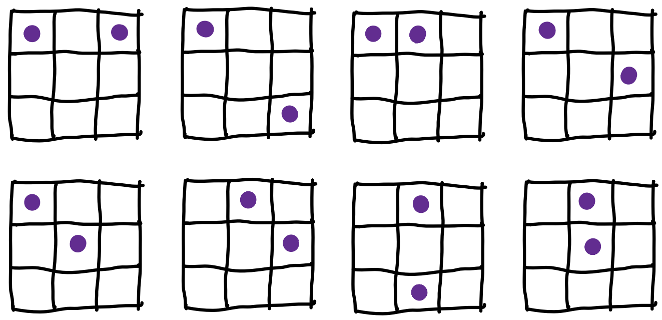
	$g=1$	$g=\circlearrowright$ (180°), $m=\lfloor \frac{n^2}{2} \rfloor$	$g=\circlearrowright,\circlearrowleft$ (90°), $m=\lfloor \frac{n^2}{4} \rfloor$	$g=\leftrightarrow,\updownarrow$, $m=n\lfloor \frac{n}{2} \rfloor$	$g=\nearrow\!\!\swarrow,\nwarrow\!\!\searrow$, $m=\frac{n^2-n}{2}$
$k=2$	$\binom{n^2}{2}$	m	0	$m+[n \text{ odd?}]\binom{n}{2}$	$m+\binom{n}{2}$
$n=2$	$\binom{4}{2}=6$	$\lfloor \frac{4}{2} \rfloor=2$	0	$2\lfloor \frac{2}{2} \rfloor=2$	$\frac{4-2}{2}+\binom{2}{2}=2$

$$\frac{1}{8}(\underbrace{6+2}_{8}+2\cdot 0+\underbrace{2\cdot 2+2\cdot 2}_{8}) = \frac{16}{8}=2$$



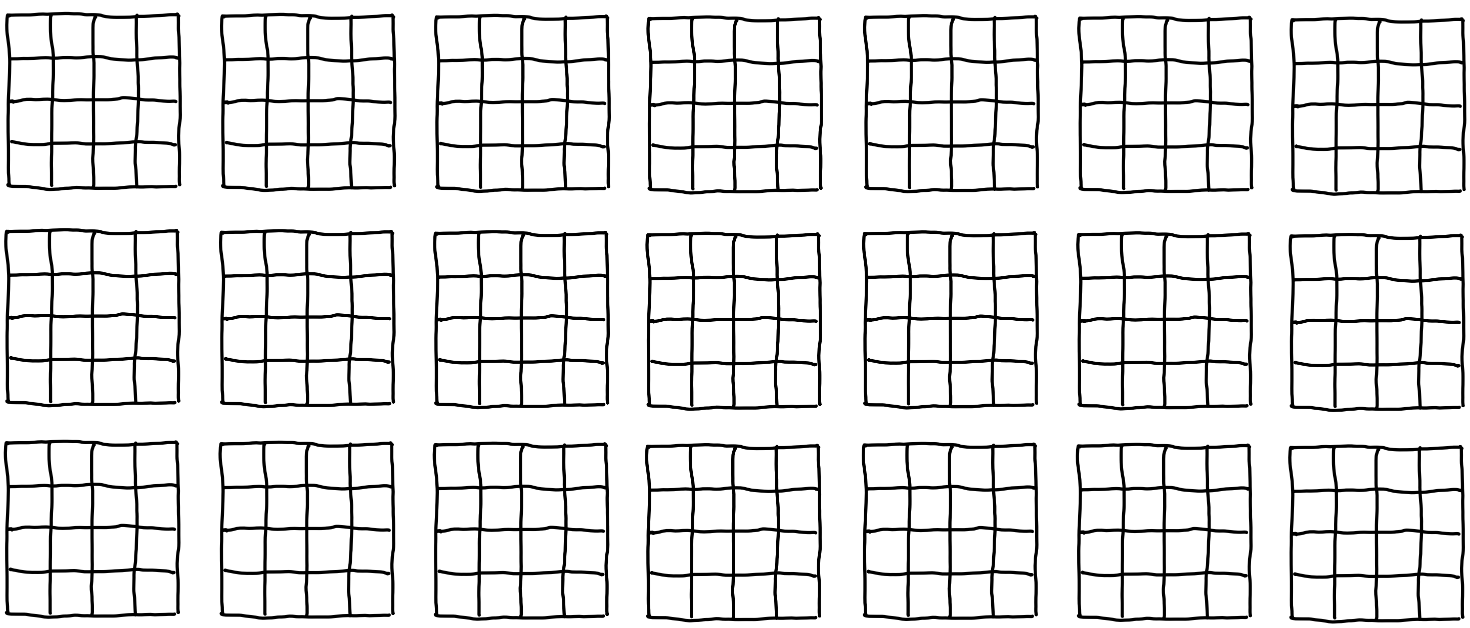
$n=3$	$\binom{9}{2}=36$	$\lfloor \frac{9}{2} \rfloor=4$	0	$3\lfloor \frac{3}{2} \rfloor+\binom{3}{2}=6$	$\frac{9-3}{2}+\binom{3}{2}=6$

$$\frac{1}{8}(\underbrace{36+4}_{40}+2\cdot 0+\underbrace{2\cdot 6+2\cdot 6}_{24}) = \frac{64}{8}=8$$



$n=4$	$\binom{16}{2}=120$	$\lfloor \frac{16}{2} \rfloor=8$	0	$4\lfloor \frac{4}{2} \rfloor=8$	$\frac{16-4}{2}+\binom{4}{2}=12$

$$\frac{1}{8}(\underbrace{120+8}_{128}+2\cdot 0+\underbrace{2\cdot 8+2\cdot 12}_{40}) = \frac{168}{8}=21$$



	$g=1$	$g=$ $m=\left\lfloor \frac{n^2}{2} \right\rfloor$	$g=$ $m=\left\lfloor \frac{n^2}{4} \right\rfloor$	$g=$ $m=n\left\lfloor \frac{n}{2} \right\rfloor$	$g=$ $m=\frac{n^2-n}{2}$
$K=3$	$\binom{n^2}{3}$	$[n \text{ odd?}]\, m$	0	$[n \text{ odd?}]\, mn$	mn
$K=4$	$\binom{n^2}{4}$	$\binom{m}{2}$	m	$\binom{m}{2}+[n \text{ odd?}]\left(m\binom{n}{2}+\binom{n}{4}\right)$	$\binom{m}{2}+m\binom{n}{2}+\binom{n}{4}$