



## Exam 1

Combinatorics, Dave Bayer, February 15-19, 2023

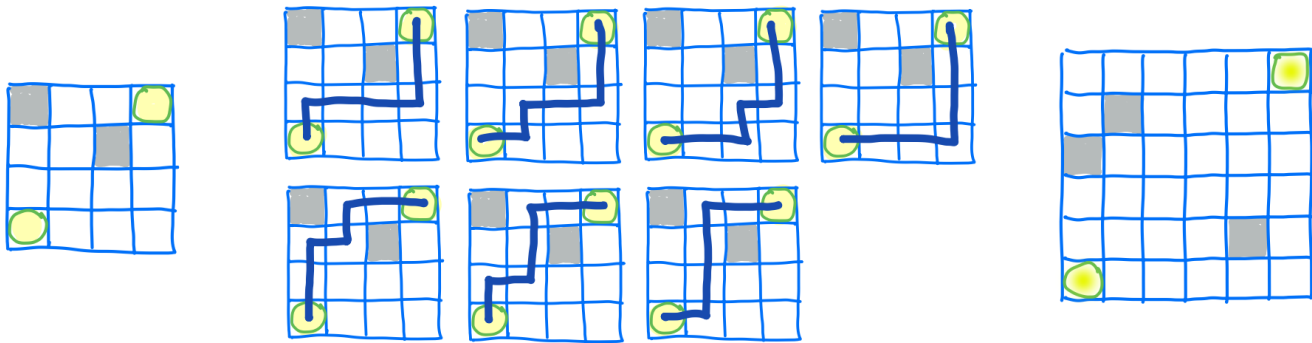
Please show all of your work. You will be graded for both your answers and your explanations. You need not complete the entire exam; some questions are intended to be challenging.

This test is open-book. You may use any resource such as my course materials, textbooks, or *The On-Line Encyclopedia of Integer Sequences*. You may not receive help from another person.

“What can you say about  $f(n)$ ?” is up to you. There might be a formula. There might be a generating function. You might notice a pattern, or recognize the sequence.

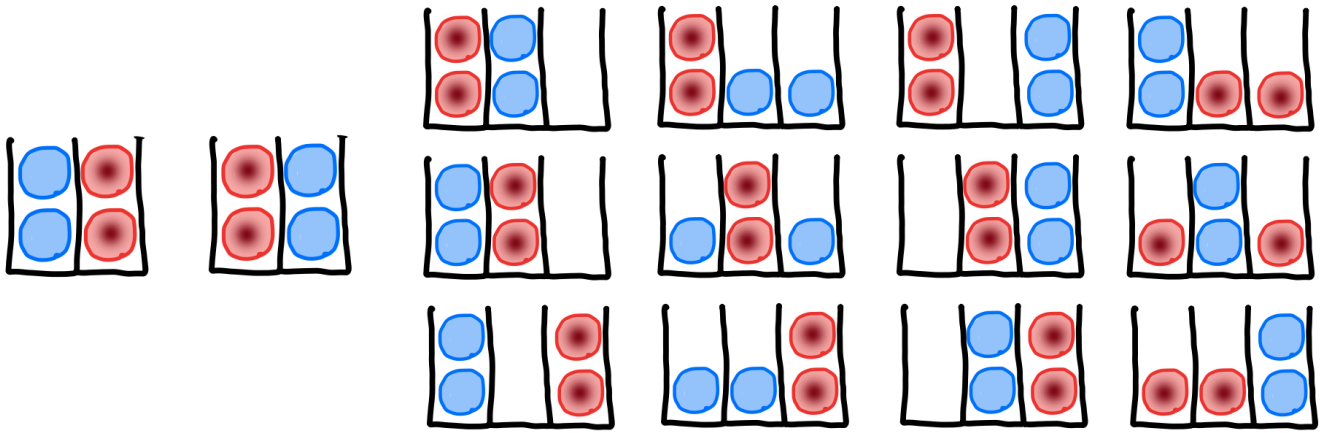
Please match your understanding of my words with the examples, and contact me if you’re concerned about any ambiguity.

[1] Moving up or to the right, for the smaller grid on the left there are seven paths between the marked corners that avoid the obstacles. For the larger grid on the right, how many paths avoid the obstacles?



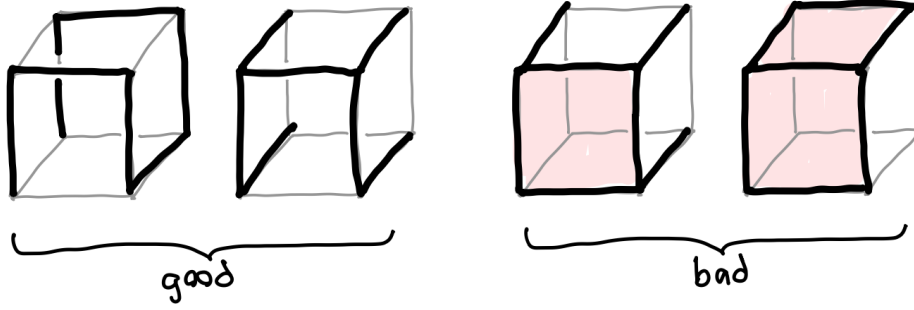


[2] Let  $f(n)$  count the number of ways of placing two red balls and two blue balls in  $n$  bins, with the restriction that no bin can contain both a red and a blue ball. As shown,  $f(2) = 2$  and  $f(3) = 12$ . What is  $f(4)$ ? What can you say about  $f(n)$ ?



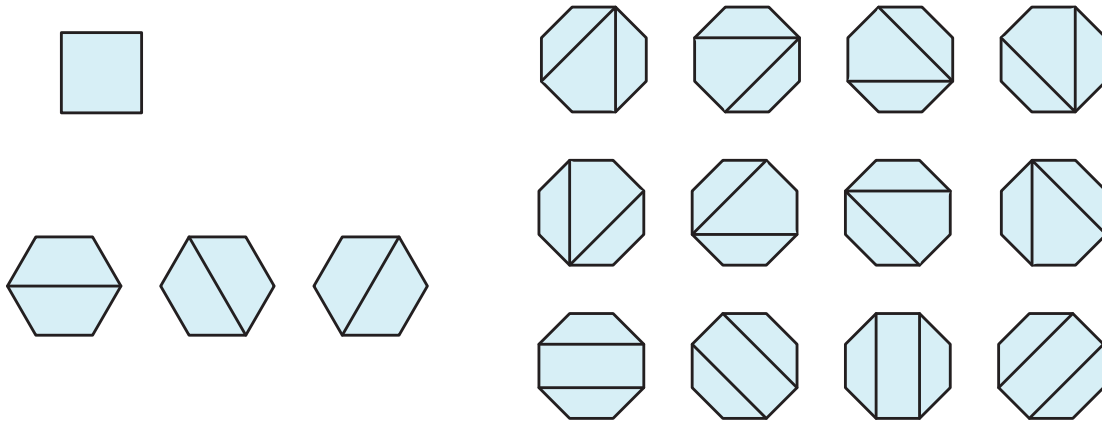


[3] How many ways can we mark seven edges of a cube, so that no square face has all four edges marked?





[4] Let  $f(n)$  count the number of ways of dissecting a  $2n$ -gon into 4-sided regions. As shown,  $f(2) = 1$ ,  $f(3) = 3$ , and  $f(4) = 12$ . What is  $f(5)$ ? What can you say about  $f(n)$ ?





[5] Let  $f(n)$  count the number of ways of tiling a  $2 \times n$  grid using  $1 \times 1$  and  $1 \times 2$  tiles, where one is allowed to leave squares untilled. As shown,  $f(1) = 5$ , and  $f(2) = 34$ . What is  $f(3)$ ? What can you say about  $f(n)$ ?

