MR848681 (87i:32031) 32G15 14H15 57R20
Harer, J. [Harer, John Lester] (1-MD); **Zagier, D. [Zagier, Don Bernard]** (1-MD)
The Euler characteristic of the moduli space of curves.
Invent. Math. **85** (1986), *no. 3,* 457–485.

The moduli space of curves or Riemann surfaces S of genus g with n punctures is denoted by $\mathcal{M}_g^n$ or $\mathcal{M}_g$ when $n = 0$. It may be expressed as the Teichmüller space $\mathcal{T}_g^n$ or $\mathcal{T}_g$ modulo the Teichmüller modular group, Mod. In the cases of interest here, $g \geq 1$ and $n = 0, 1$. Mod is generally the same as the mapping class group Γ_g^n of S—for $n = 0, 1$ the only exceptions are $g = 1$, $n = 1$ or $g = 2$, and $n = 0$. In the exceptional cases, Γ_g^n is the extension of Mod by the common hyperelliptic involution on all surfaces in $\mathcal{T}_g^n$. C. T. C. Wall[Proc. Cambridge Philos. Soc. **57** (1961), 182–184; MR0122853] introduced a rational Euler characteristic $\chi(G)$ for groups which, in the present situation, is the Euler characteristic $\chi(X)/m$, where $\chi(X)$ is the usual Euler characteristic of a finite sheeted cover of $\mathcal{M}_g^n$ by a manifold given as $\mathcal{T}_g^n$ modulo a fixed point free finite index subgroup Γ of Γ_g^n. m is then the index of Γ in Γ_g^n. Quillen showed how to compute $\chi(G)$ when it acts on a simplicial complex.

The main result in this paper is that $\chi(\Gamma_g^1) = \zeta(1-2g)$ $(= -B_{2g}/2g)$. Here ζ is the Riemann zeta function and B denotes the Bernoulli number. The simplicial complex used is derived from a construction of Riemann surfaces due to Strebel. The Quillen-style computation is quite complicated, both combinatorially and analytically.

Using results of K. Brownand still more combinatorics, a formula is derived for the usual Euler characteristics e of both Γ_g^1 and Γ_g. The formulas are too complicated to reproduce and explain here.

Recently, Penner has announced an alternative method for finding $\chi(\Gamma_g^n)$ for $n > 0$.

{See also the following review.} *William Abikoff*