Elements of Grothendieck's life and work

Caleb Ji

Four stages of life

Caleb Ji

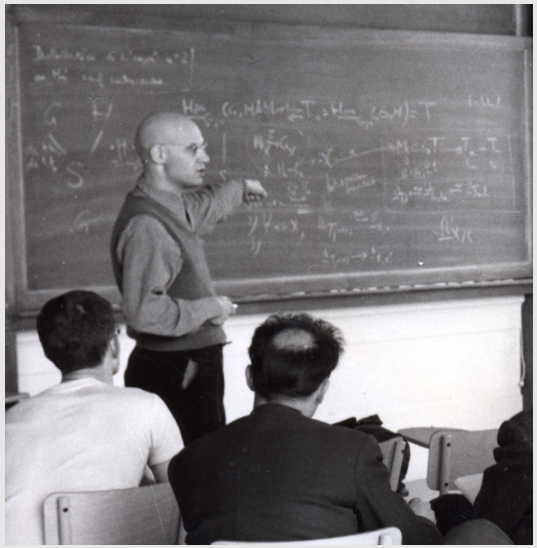
Part 1: Upbringing (1928–1949)

Part 2: Mathematics (1949–1970)

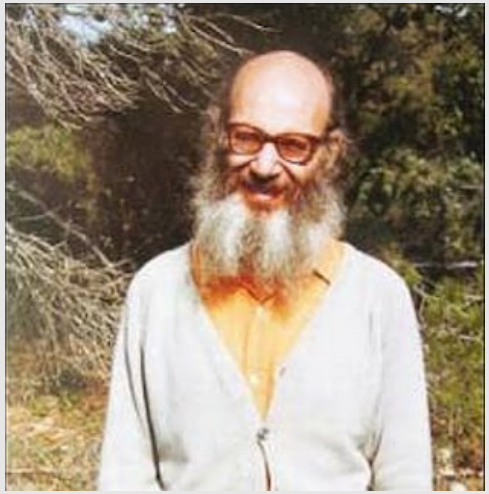
Part 3: Freedom (1970–1991)

Part 4: Hermitage (1991–2014)

(a) 1928-1949



(b) 1949-1970



(c) 1970-1991

(d) 1991-2014

Sascha Schapiro (1890 – 1942)

- Born to a well-off middle-class Jewish family in Ukraine
- Gives up his family to join an anarchist revolution against the Tsar at fourteen
- Captured at 16, sentenced to life in prison, rescued by and joins the Russian Revolution in 1917
- Allied with Makhno, pursued by Bolsheviks, emigrates to Berlin, supporting himself with photography

Sascha Schapiro (1890 – 1942)

- Met Hanka Grothendieck in Berlin anarchist circles
- Emigrate together to France to escape Nazism
- Fights in the Spanish Revolution, violently crushed by Franco
- Returns to France, deported to Auschwitz in 1942

Figure: Sascha and Hanka

Hanka Grothendieck (1900 - 1957)

- Born to a bourgeois family in Hamburg, broke away as a teenager
- Actress, nomad, editor, writer
- Marries Alf Raddatz ̃1921: pilot, communist, vagabond
- Meets Sasha, supports the Spanish revolutionaries, sent to internment camp in France during WWII
- Lives close to her son in later years. Writes an unfinished autobiography *Eine Frau*

Sascha and Hanka

- Well-read despite material difficulties, aspiring writers
- Extremely aggressive personalities
- All relationships highly troubled

(AG, *La Clef des Songes*)

"And during the long years after the abrupt end of an immense epic adventure, he carried within him the work to be accomplished – a rich fresco of faith, hope and suffering, of laughter and tears and blood that has been shed, crowded and vast like his own indomitable life, and keen like a song of liberty [...] And my mother also had blessed gifts, which predestined her for great things. But they chose to neutralize each other in an endless passionate confrontation [...] "

Childhood

- Left in Hamburg with the Heydorns, his foster family at the age of 5
- Reunited with parents in Paris at 11
- Father murdered in Auschwitz, sent to internment camps with his mother to live in poverty
- Worked the grape harvest, enjoyed school, especially math

Student at Montpellier

(AG, R&S)

What I found most unsatisfactory in my mathematics textbooks was the absence of any serious attempt to tackle the meaning of the idea of the arc-length of a curve, or the area of a surface or the volume of a solid. I resolved therefore to make up for this defect once I found time to do so. In fact I devoted most of my energy to this when I became a student at the University of Montpellier, between 1945 and 1948.

Arrival in Paris (1948)

- Received a scholarship to study in Paris thanks to André Magnier
- Attended the famous Cartan seminar

(AG, R&S)

I discovered the existence of a mathematical world when I got to Paris in 1948, at the age of 20, with not much more in my thin suitcase than a bachelor's degree in science and a closely-written manuscript using both sides of the paper and without margins (paper was expensive!), representing three years of solitary reflections on what (I later learned) was known as "measure theory" or "the Lebesgue integral". Since I had never met another, I believed until the day I arrived in the capital, that I was the only person in the world to "do mathematics", the only mathematician.

The Cartans

- Élie Cartan (1869 – 1951) greatly developed Lie theory, differential equations, and differential geometry
- Henri Cartan (1904 – 2008) contributed to complex analytic geometry, algebraic topology, and homological algebra
- Cartan seminar (1948 – 1964) covered much of the cutting-edge research at the time

Figure: Élie Cartan

Figure: Henri Cartan

Nicolas Bourbaki

The Bourbaki group: a group of mostly French mathematicians dedicated to properly building the foundations of mathematics, founded by Cartan, Weil, Dieudonné, Delsarte, Chevalley

Figure: Claude Chevalley

Figure: Andre Weil

PhD in Nancy: functional analysis (1949-1953)

- Worked with Jean Dieudonné and Laurent Schwartz
- Solved all 14 of their problems on locally convex topological vector spaces
- Develops the concept of nuclear spaces in his thesis, proceeding to revolutionize functional analysis

Figure: Jean Dieudonné

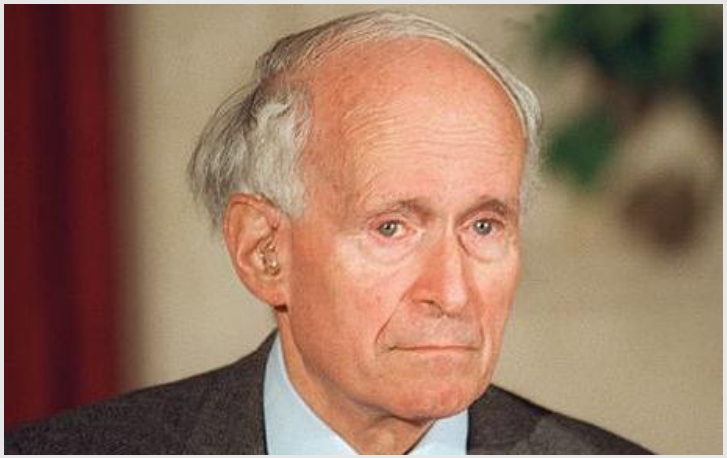
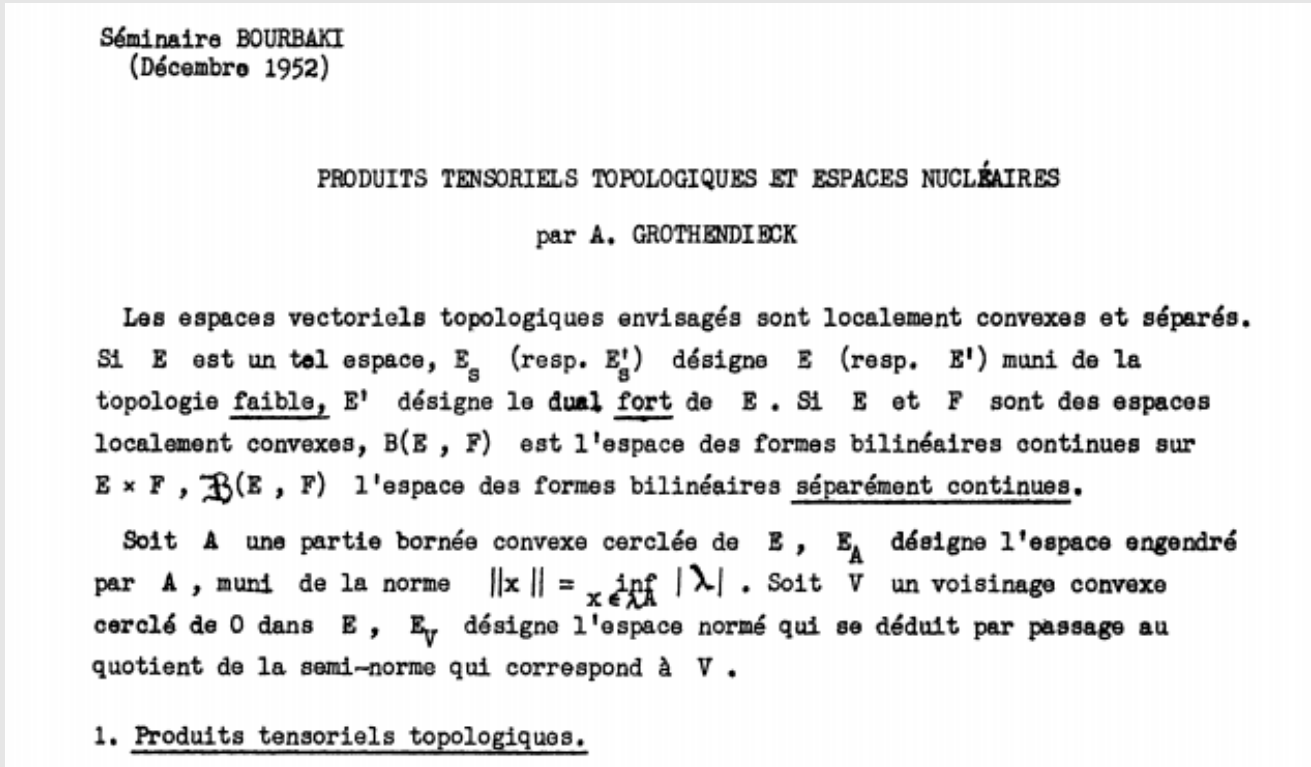


Figure: Laurent Schwartz

PhD in Nancy: functional analysis (1949-1953)

(Jean Dieudonné)

"When, in 1953, it was time to grant him a doctor's degree, it was necessary to choose from among six papers he had written, any one of which was at the level of a good dissertation."



Séminaire BOURBAKI
(Décembre 1952)

PRODUITS TENSORIELS TOPOLOGIQUES ET ESPACES NUCLÉAIRES

par A. GROTHENDIECK

Les espaces vectoriels topologiques envisagés sont localement convexes et séparés. Si E est un tel espace, E_s (resp. E'_s) désigne E (resp. E') muni de la topologie faible, E' désigne le dual fort de E. Si E et F sont des espaces localement convexes, B(E, F) est l'espace des formes bilinéaires continues sur $E \times F$, $\mathfrak{B}(E, F)$ l'espace des formes bilinéaires séparément continues.

Soit A une partie bornée convexe cerclée de E, E_A désigne l'espace engendré par A, muni de la norme $\|x\| = \inf_{x \in \lambda A} |\lambda|$. Soit V un voisinage convexe cerclé de 0 dans E, E_V désigne l'espace normé qui se déduit par passage au quotient de la semi-norme qui correspond à V.

1. Produits tensoriels topologiques.

Figure: Grothendieck's thesis

A new life in Brazil: 1953-1955

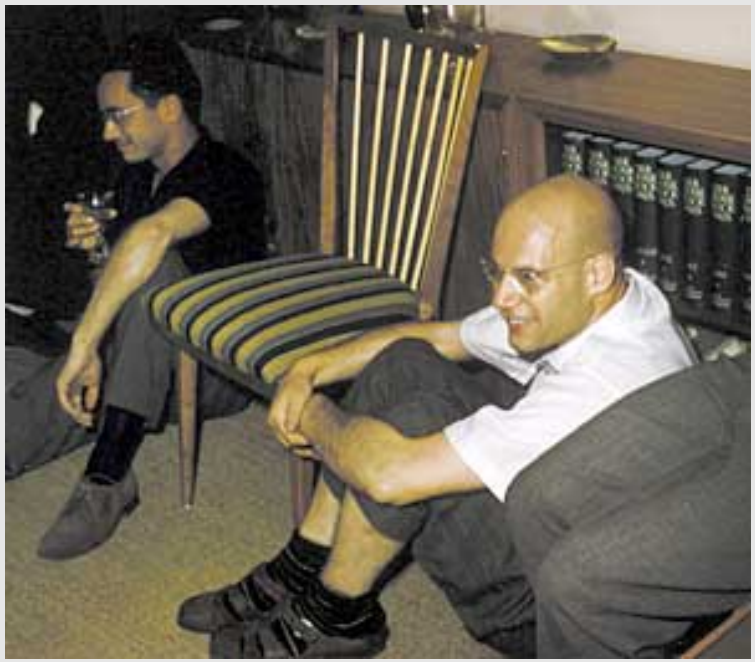
- Spent two years in São Paulo
- Subsisted on milk, bread, and bananas
- Frightful family fiasco on trip to Paris
- Tying up loose ends, beginning to work on topology

(AG)

"So now I can finally leave the field of topological vector spaces with no regrets, and start seriously working on algebraic topology."

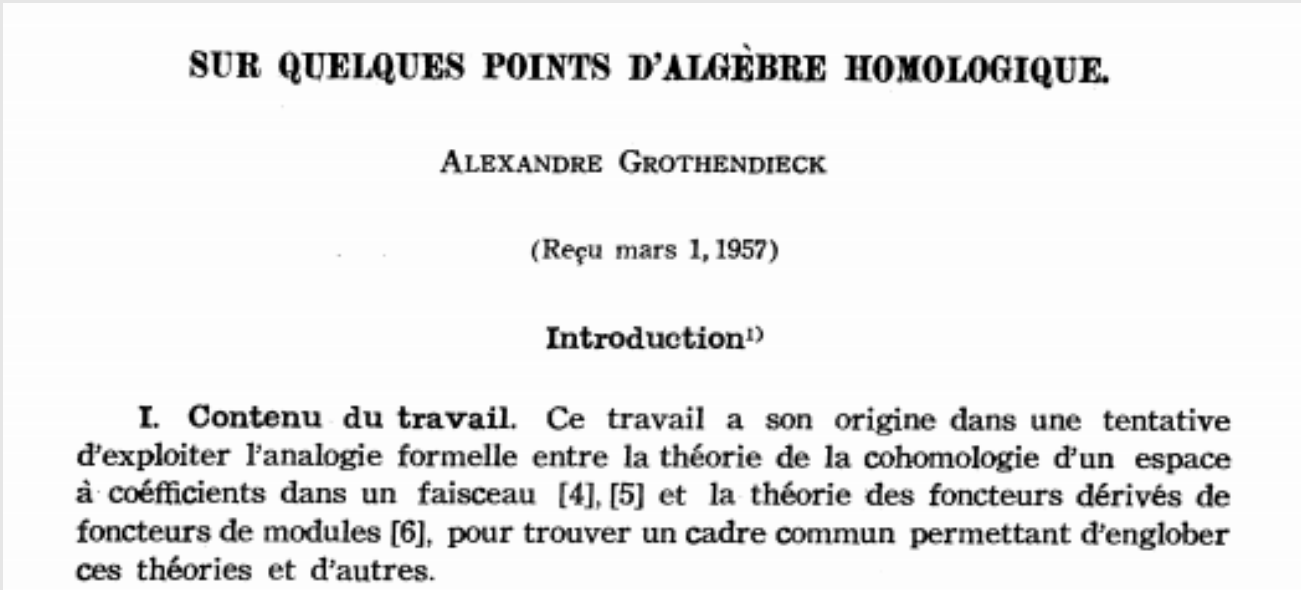
New work in Kansas: 1955

- Visiting research associate professor at the University of Kansas in 1955
- Wrote "A General Theory of Fibre Spaces with Structure Sheaf"
- Wrote "Sur quelques points d'algèbre homologique"
- Began correspondence with Jean-Pierre Serre



The Tôhoku paper

- *Sur quelques points d'algèbre homologique*, published in Tôhoku, 1957
- Grew from Grothendieck's attempt to reconstruct the Cartan-Eilenberg book on homological algebra
- Introduced abelian categories
- Gave a general framework for cohomology through derived functors, which worked for sheaves



SUR QUELQUES POINTS D'ALGÈBRE HOMOLOGIQUE.

ALEXANDRE GROTHENDIECK

(Reçu mars 1, 1957)

Introduction[1)]

I. Contenu du travail. Ce travail a son origine dans une tentative d'exploiter l'analogie formelle entre la théorie de la cohomologie d'un espace à coéfficients dans un faisceau [4], [5] et la théorie des foncteurs dérivés de foncteurs de modules [6], pour trouver un cadre commun permettant d'englober ces théories et d'autres.

Return to France

- Obtains a position at the CNRS, moves back to Paris in 1956
- Continued to work deeper in topology and algebraic geometry

(Armand Borel)

I was sure something first-rate would come out of him. But then what came out was even much higher than I had expected. It was his version of Riemann-Roch, and that's a fantastic theorem. This is really a masterpiece of mathematics.

Grothendieck-Riemann-Roch

Caleb Ji

Part 1: Upbringing (1928–1949)

Part 2: Mathematics (1949–1970)

Part 3: Freedom (1970–1991)

Part 4: Hermitage (1991–2014)

Theorem (Riemann-Roch)

For a compact Riemann surface with canonical divisor K and genus g, for any divisor D we have

$$l(D) - l(K - D) = \deg(D) - g + 1.$$

Theorem (Grothendieck-Riemann-Roch)

Let X, Y be locally of finite type over S. Let $\mathcal{F}$ be a finite locally free sheaf on X of rank r. Let $f : X \to Y$ be a proper smooth morphism. Then
$ch(f_!\mathcal{F}^\bullet)td(Y) = f_*(ch(\mathcal{F}^\bullet)td(X))$.

Grothendieck-Riemann-Roch

Caleb Ji

Part 1: Upbringing (1928–1949)

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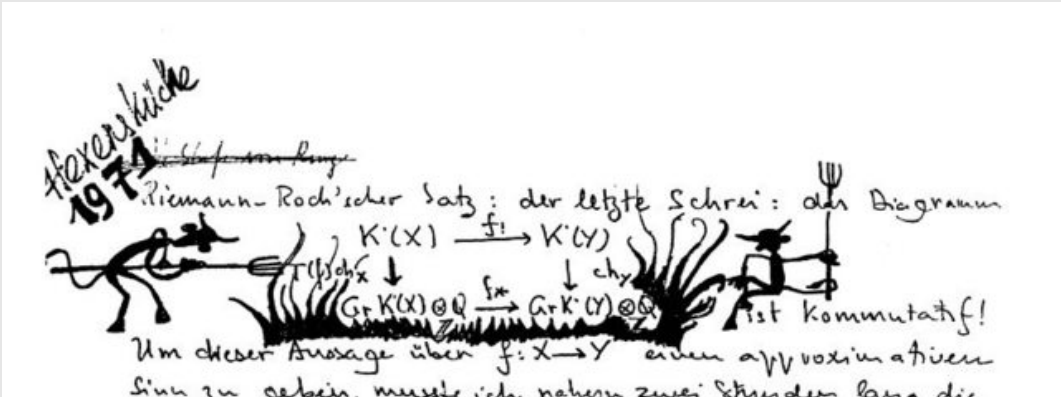
(AG)

"Witches Kitchen 1971. Riemann-Roch Theorem: The final cry: The diagram is commutative! To give an approximate sense to the statement about $f : X \to Y$, I had to abuse the listeners' patience for almost two hours. Black on white (in Springer lecture notes) it probably takes about 400, 500 pages. A gripping example of how our thirst for knowledge and discovery indulges itself more and more in a logical delirium far removed from life, while life itself is going to Hell in a thousand ways and is under the threat of final extermination. High time to change our course!"



1957-1958: the beginning of a new era

(AG, R&S)

"1957 was the year in which I began to develop the theme "Riemann-Roch" (Grothendieck version) - which almost overnight made me into a big "movie star". It was also the year of my mother's death and thereby the inception of a great break in my life story. They figure among the most intensely creative years of my entire life, not only in mathematics. I'd worked almost exclusively in mathematics for 12 years. In that year there was the sense that I'd perhaps done what there was to do in mathematics and that it was time to try something else.

1957: death of Hanka

(AG, La Clef des Songes)

"It was given to me to be with her in the last weeks of her life, to care for her and see her die. And also, in these final weeks, to see dissipate as if it had never been, the arid and bitter despair in which it had been maintained for the last five years. So his death came as the unexpected resolution of such accumulated tension that I believe it would have broken me if my mother hadn't died reconciled, loving and at peace.

- Resolves to become a writer, to complete Hanka's *Eine Frau*
- Ends up back in mathematics for 12 more years!

1958: a new era in algebraic geometry

(AG, R&S)

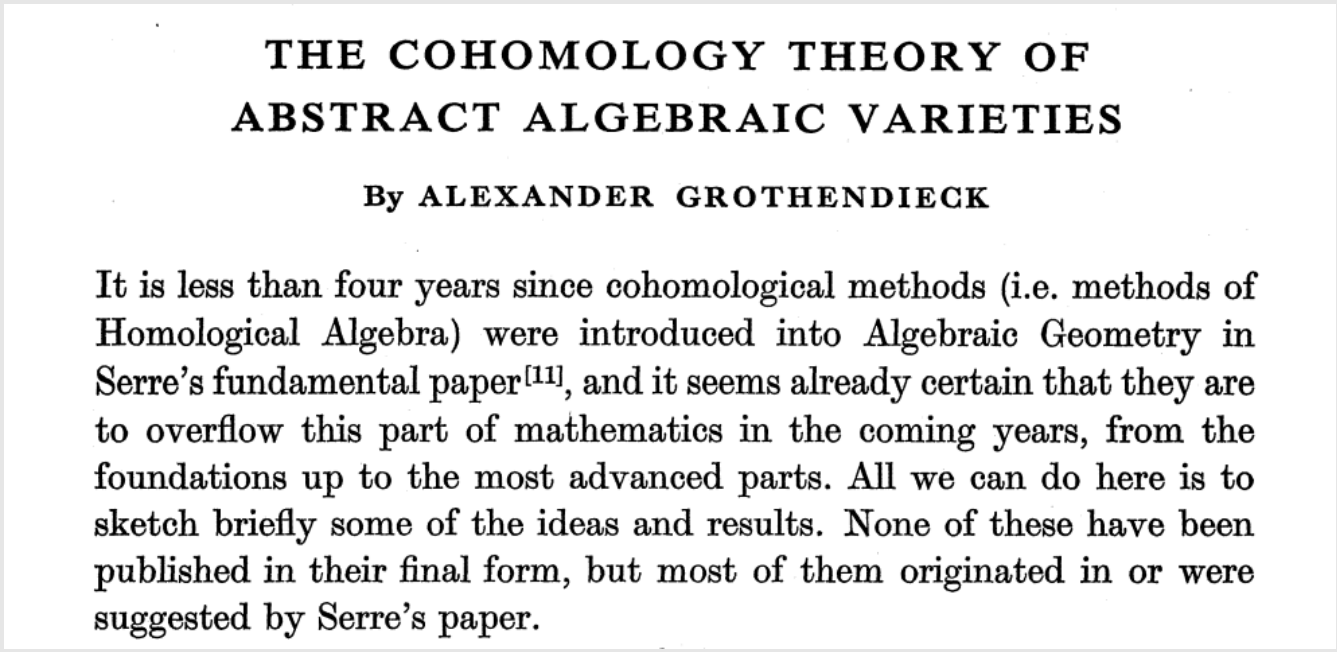
"The following year (1958) is probably the most fertile of all my years as a mathematician. This was the year which saw the birth of the two central themes of the new geometry through the launching of the theory of schemes (the subject of my paper at the International Congress of Mathematicians at Edinborough in the summer of that year) and the appearance of the concept of a "site", a provisional technical form of the crucial notion of the topos."

Grothendieck's family

- Affair with Aline Driquert, resulting in a son: Serge Grothendieck
- Married to Mireille Dufour, resulting in three children: Johanna, Alexandre, Matthieu Grothendieck
- Marriage lasted around 1957 – 1970, though not formally divorced till 1981
- Later affair with Justine Bumby, resulting in a son: John Grothendieck

The 1958 ICM address

- Inspired by Serre's FAC, announces the introduction of cohomology into algebraic geometry
- Mentions the Weil conjectures as a future goal
- Introduced schemes and gives immediate applications



THE COHOMOLOGY THEORY OF ABSTRACT ALGEBRAIC VARIETIES

By ALEXANDER GROTHENDIECK

It is less than four years since cohomological methods (i.e. methods of Homological Algebra) were introduced into Algebraic Geometry in Serre's fundamental paper[11], and it seems already certain that they are to overflow this part of mathematics in the coming years, from the foundations up to the most advanced parts. All we can do here is to sketch briefly some of the ideas and results. None of these have been published in their final form, but most of them originated in or were suggested by Serre's paper.

Figure: Grothendieck's address

The 1958 ICM address

- Cohomological topics outlined:
  1. Weil cohomology for algebraic varieties
  2. Cohomology of coherent sheaves
     1. Finiteness theorems
     2. Duality theorems
     3. Riemann-Roch theorems
     4. Theorems on abelian varieties
  3. Applications to local algebra
- Also emphasizes the transition from varieties to schemes and even morphisms of schemes, the relative point of view

Schemes

Definition (schemes)

Given any commutative ring A, define the affine scheme $\operatorname{Spec} A$ to be the locally ringed space consisting of the prime ideals of A equipped with the topology of closed sets being those containing a fixed subset of A. The structure sheaf is defined by $O_{\operatorname{Spec} A}(D(f)) = A_f$. A scheme is obtained by gluing affine schemes along open subsets.

(AG, R&S)

"The very notion of a scheme has a childlike simplicity - so simple, so humble in fact that no one before me had the audacity to take it seriously."

Schemes

- Grothendieck also develops the functorial point of view, which (by Yoneda's lemma) characterizes a scheme X by the functor $h_X : \mathrm{Sch}^{\mathrm{opp}} \to \mathrm{Set}$

$$h_X(T) = \mathrm{Mor}(T, X).$$

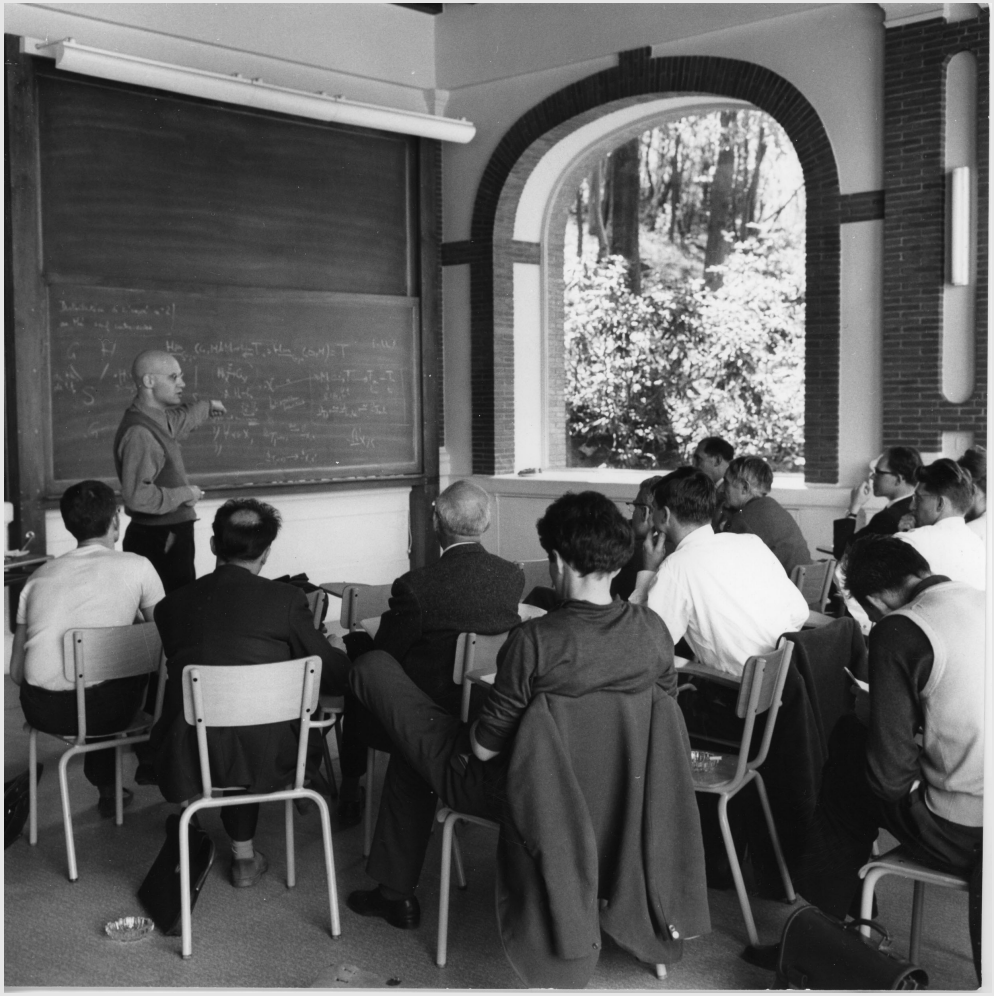
- In fact, one can restrict to affine schemes, and thereby define schemes by imposing appropriate conditions on functors $F : \mathrm{Rings} \to \mathrm{Set}$.
- This point of view translates geometric properties into categorical ones, and is used to construct new types of geometric spaces, including algebraic stacks (used heavily in moduli theory).

The Institut des hautes études scientifiques (IHES)

- Léon Motchane, businessman and mathematician, secures funds for the IHES
- Modeled after the Institute for Advanced Study (IAS) in Princeton, goal to retain and support French scientific research
- Located in Bures-sur-Yvette, a Parisian suburb
- Grothendieck and Dieudonné recruited as the first professors

Professor at the IHES (1958–1970)

- Known as the Golden Age
- Creates math 12 hours a day, 7 days a week
- Leads his legendary SGA seminars



EGA and SGA

EGA (with J. Dieudonné)

1. Le langage des schémas
2. Étude globale élémentaire de quelques classes de morphismes
3. Étude cohomologique des faisceaux cohérents
4. Étude locale des schémas et des morphismes de schémas

SGA (with Raynaud, Demazure, Artin, Verdier, Illusie, Berthelot, Deligne, Katz)

1. Revêtements étales et groupe fondamental
2. Cohomologie locale des faisceaux cohérents et théorèmes de Lefschetz locaux et globaux
3. Schémas en groupes
4. Théorie des topos et cohomologie étale des schémas
5. Cohomologie l-adique et fonctions L
6. Théorie des intersections et théorème de Riemann-Roch
7. Groupes de monodromie en géométrie algébrique

Mathematical opus

Through 1970, includes:

1. Topological tensor products and nuclear spaces
2. Derived categories, the 6 operations
3. Grothendieck-Riemann-Roch, K-theory
4. Schemes
5. Topos theory
6. Étale cohomology and l-adic cohomology
7. Motives, motivic Galois groups
8. Crystalline Cohomology, de Rham and Hodge coefficients
9. Topological algebra, ∞-stacks

and afterwards:

10. Mediated topology
11. Anabelian geometry, Galois-Teichmüller theory
12. Schematic or arithmetic Viewpoints for regular polyhedra

Weil conjectures

Let X be a projective non-singular algebraic variety defined over $\mathbb{F}_q$. The zeta function $\zeta(X, s)$ is defined as

$$\zeta(X, s) := \sum_{m \geq 1} \frac{N_m}{m} q^{-ms},$$

where N_m is the number of points of X defined over $\mathbb{F}_{q^m}$.

Theorem (Weil conjectures)

1. $\zeta(X, s)$ *is a rational function of* $T = q^{-s}$.
2. $\zeta(X, n - s) = \pm q^{\frac{nE}{2} - Es} \zeta(X, s)$ *where* E *is the Euler characteristic of* X.
3. $|\alpha_{i,j}| = q^{i/2}$ *for* $1 \leq i \leq 2n - 1$ *and all* j.
4. *If* X *is a (good) "reduction* (mod p)*" of a non-singular projective variety* Y *defined over a number field embedded in the field of complex numbers, then the degree of* P_i *is the* i*th Betti number of the space of complex points of* Y.

Weil conjectures

(AG, R&S)

“ These utterly astounding conjectures allowed one to envisage, for these new ” discrete varieties” (or ”spaces”), the possibility for certain kinds of constructions and arguments which up to that moment did not appear to be conceivable outside of the framework of [topological spaces].”

(AG, R&S)

”The panorama that opened up before me, which I was obliged to make the effort to scrutinize and capture, greatly surpassed in scope and in depth the hypothetical needs for proving these conjectures, or indeed all the results that would follow from them.”

Motives

(AG, R&S)

"In contradistinction to what one finds in ordinary topology, one finds oneself in the presence of a disconcerting abundance of differing cohomological theories. One had the impression that, in a sense that should be taken rather flexibly, all of these theories "boiled down" to the same one, that they "gave the same results"."

(AG, R&S)

"This idea was developed, on the fringes of more fundamental and urgent tasks, under the name of the "theory of motives", or of "philosophy (or "yoga") of the "motives", through the years 1963-69. It's a theory of a fascinating structural richness, a large part of which remains purely conjectural."

The Fields medal

- Wins Fields medal in 1966
- Refuses to go to Moscow to accept it in protest over human rights violations
- Donates prize money, auctions off medal to the Red Cross of Vietnam

(ICM citation)

Built on work of Weil and Zariski and effected fundamental advances in algebraic geometry. He introduced the idea of K-theory (the Grothendieck groups and rings). Revolutionized homological algebra in his celebrated "Tohoku paper."

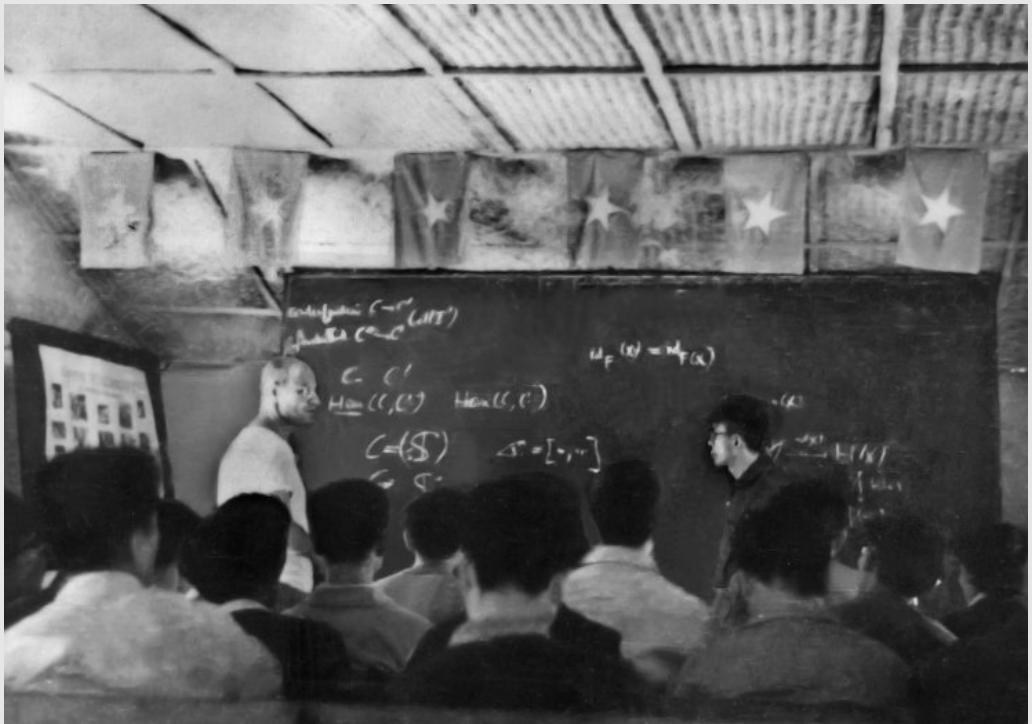
Some PhD students

- Demazure (algebraic groups)
- Illusie (cotangent complex, deformation theory)
- Verdier (derived categories)
- Deligne (mixed Hodge theory)
- Giraud (non-abelian cohomology)
- Berthelot (crystalline cohomology)

Trip to Vietnam

- Travels to Vietnam in November 1967
- Lectures on requirements for scientific research, schemes, homological algebra, sheaf theory, and the Weil conjectures

"They have confidence in themselves, and that is the best reason for us to have confidence in them and in their struggle on all fronts, cultural as well as economic and military."



Hoàng Xuân Sính

(AG)

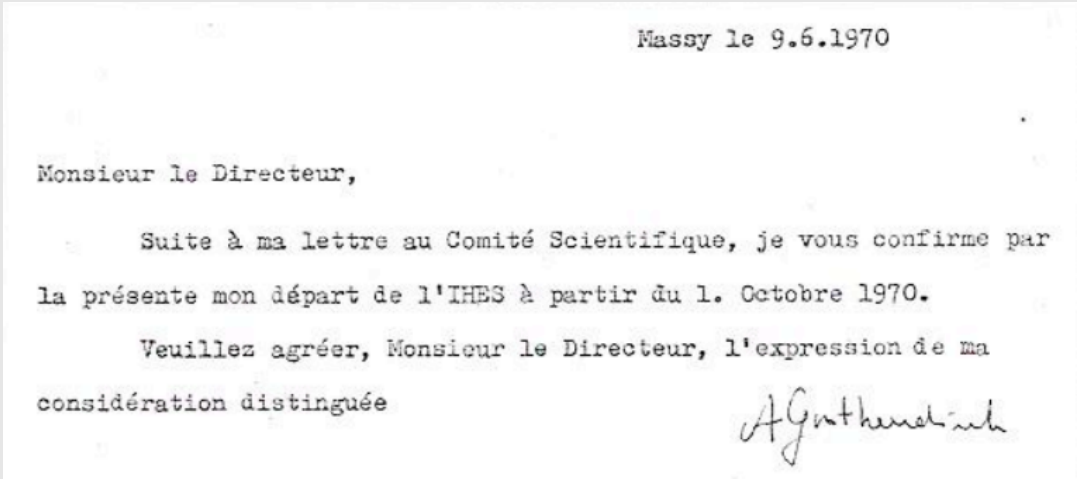
" During the war her working conditions were particularly difficult, and her contact with me was restricted to an intermittent correspondence. She was able to come to France in 1974/75 [...] and earned her doctoral degree with this thesis in Paris."

May ’68

- 1968 was a year of great left-wing, anti-war, and civil rights protests around the world
- In France, students and workers joined in a nationwide strike, reaching its peak in May ’68
- Rapidly dissolved after suppression and an election
- Left a deep impression on Grothendieck; his full commitment to mathematics began to wane

1970: The great turning point

- Grothendieck discovers that a small part of IHES funding came from the military
- The final push behind Grothendieck's decision to break with the mathematical establishment, under enormous internal personal forces



Massy le 9.6.1970

Monsieur le Directeur,

Suite à ma lettre au Comité Scientifique, je vous confirme par la présente mon départ de l'IHES à partir du 1. Octobre 1970.

Veuillez agréer, Monsieur le Directeur, l'expression de ma considération distinguée

A Grothendieck

Dear Sir, Following my letter to the Scientific Committee, I herewith confirm my departure from the IHES starting 1 October 1970. Please accept, Sir, the expression of my distinguished consideration. A. Grothendieck

Mathematical posts held post-1970 in France

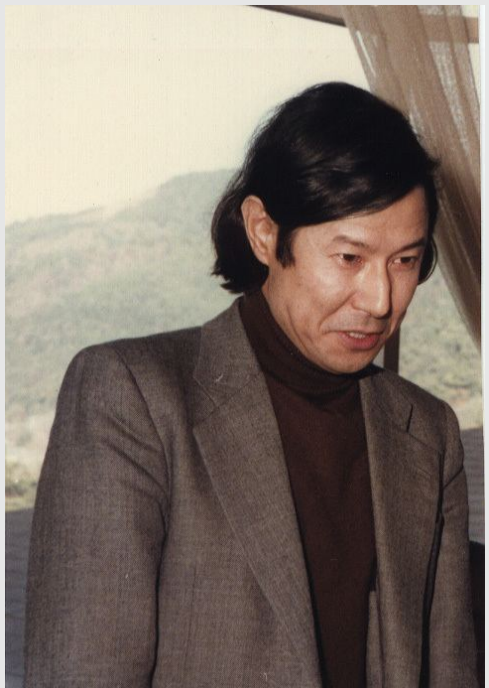
- Guest professor at Collège de France (1970-1972)
- Visiting professor at the University of Orsay (1972-1973)
- Professor at the University of Montpellier (1973-1988)
- CNRS researcher (1984-1988)

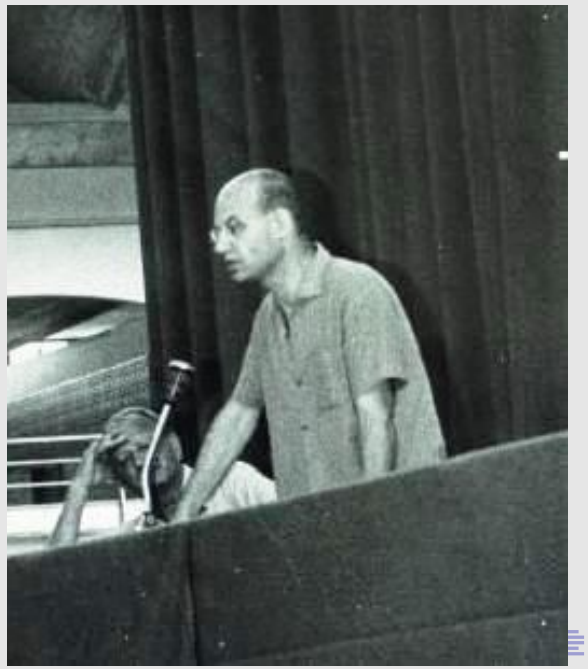
1970 ICM

Heisuke Hironaka wins the 1970 Fields medal for resolution of singularities in characteristic 0.

(AG, footnote to laudatio)

"This is all the more true since the development of algebraic geometry, and everything else as well, will come to an end if our species disappears in the coming decades – a possibility that today seems more and more likely."





Survivre et Vivre

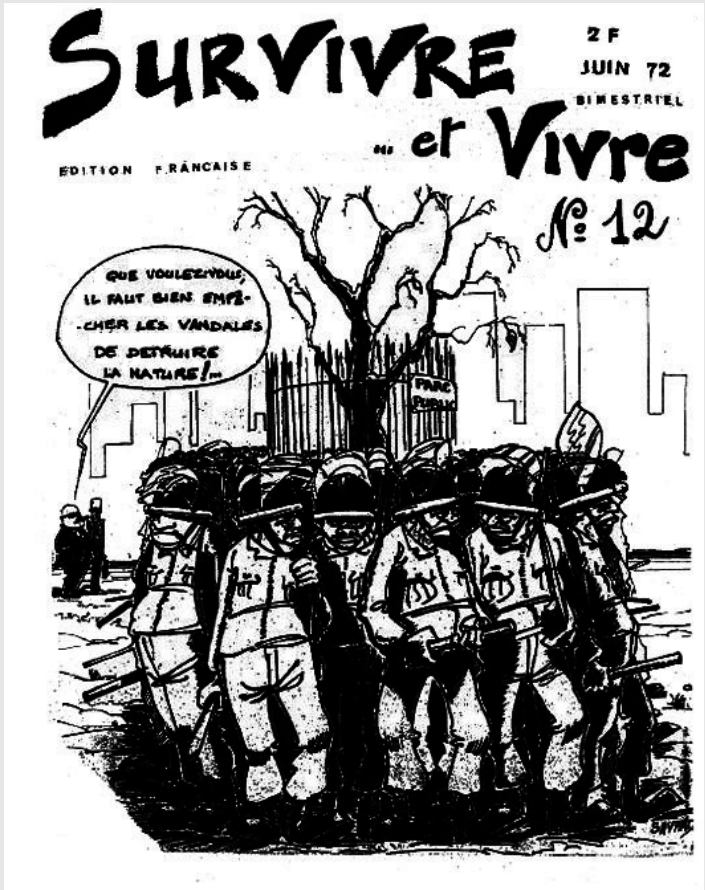
- Born on July 20, 1970 from a conference in Montreal
- Environmental, ecological goals, and political goals
- Founding members include AG, Claude Chevalley, Pierre Samuel.

(Mission statement)

"Fight for the survival of the human race and for all life, which is endangered due to the ecological imbalance caused by the industrial society of today (pollution and destruction of the environment and natural resources), and by military conflicts and the threat of military conflict."

Survivre et Vivre

- Published a bulletin, 1970 – 1973, on themes such as ecology, scientism, the military-industrial complex, nuclear power, pollution, consumerism, etc.
- Organized conferences, meetings, public discussions



Survivre et Vivre

- Grothendieck tries to recruit scientists, due to both his perception of their responsibility and his status among them
- Attends mathematical conferences to give environmental talks, to recruit, and also simply to agitate
- Tours universities, schools, meets with local movements all over the place in both France and abroad
- e.g. Invited to CERN to lecture on "Shall we continue with scientific research?"

Trips to America

- 1/1971 – 3/1971: invited professor at Queens University. Trudges through the Canadian winter barefoot in sandals, lectures on group schemes and ecology, supports nearby Native American activism
- 3/1971 – 4/1971: *Survivre* talks at 21 universities in 7 weeks, including Hamilton, Buffalo, Rochester, Stanford, Berkeley, Los Angeles, Princeton
- 5/1972 – 7/1972: Teaches at SUNY, visits Fordham, Rutgers, Brown, Albany, University of Massachusetts, Stony Brook to talk about *Survivre*
- Meets Justine Skalba at Rutgers, who follows him back to France
- Summer 1973: Lecturing up to seven hours a day in Buffalo on algebraic geometry and topos theory

Commune life

- In late 1972, founds a commune in Châtenay-Malabry along with Justine
- Gradually withdraws from *Survivre* activities
- Commune collapses in 1973 due to complicated personal relationships and lack of funds, as Grothendieck goes to Buffalo

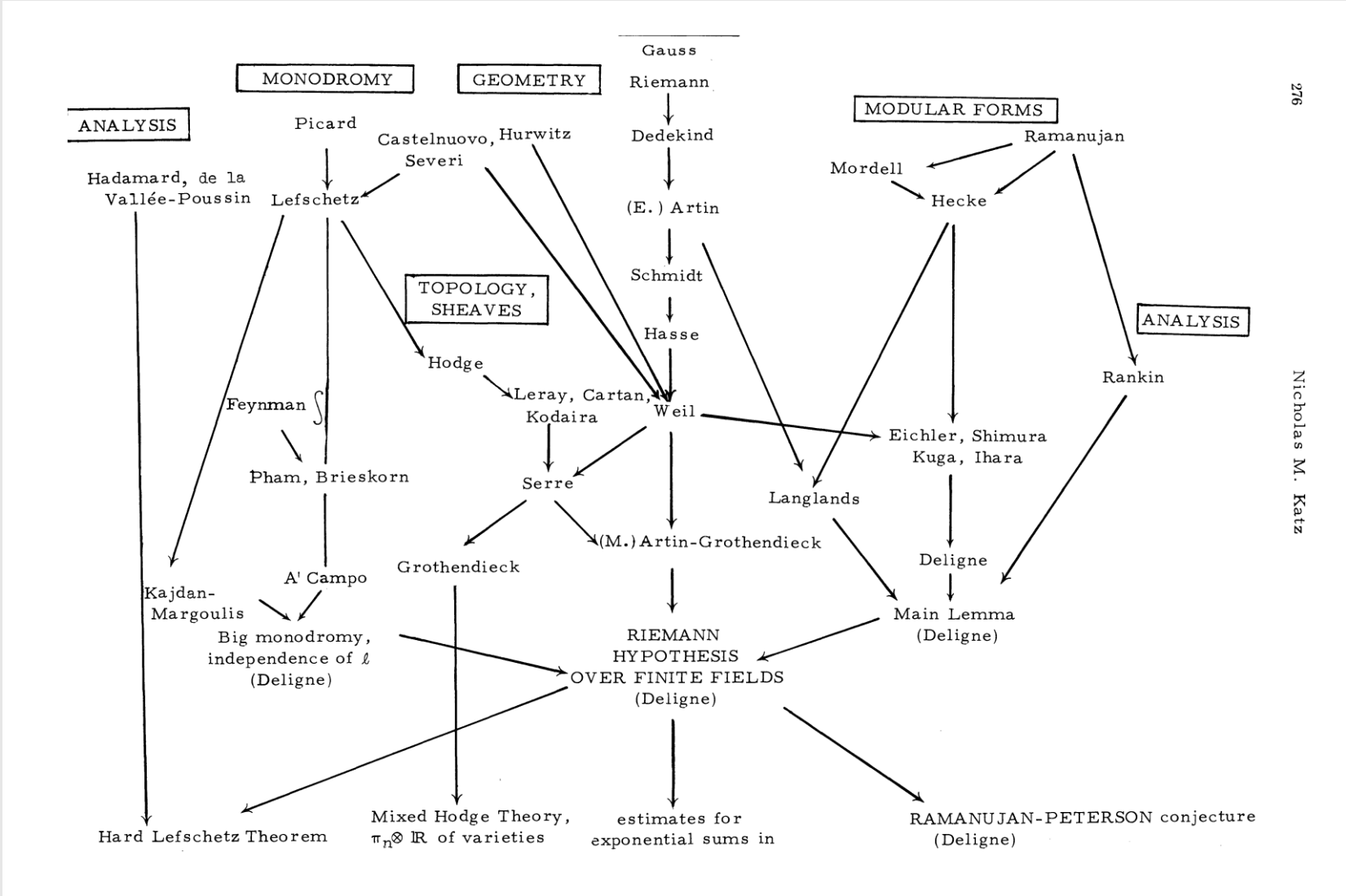
(AG, La Clef des Songes)

'Although I committed myself with the utmost energy, my reflections only touched the periphery of my existence. No doubt because I dimly perceived this, I gradually withdrew during the year 1972 from anti-militaristic, ecological and "counter-cultural" activities, since I felt that we had reached a point where we were stuck in routine engagement instead of integrating ourselves into a bigger movement ... "

Proof of the Weil conjectures: 1973

"If I had done it using motives, he would have been very interested, because it would have meant that the theory of motives had been developed. Since the proof used a trick, he did not care." - Pierre Deligne

Proof of the Weil conjectures: 1973



Life in Villecun: 1973-1979

- Moved to Villecun, a small village in France
- Justine leaves soon after with their son
- Grothendieck takes up a professorship at Montpellier

(Winfried Scharlau)

[H]e was cheerful, he brought wine, fruit, vegetables or nuts to the common meals, he sang, helped with manual jobs, was infinitely generous (in the circle of his friends he was often called "the bank"), and his house was open to everyone; he was (at least for some years) deeply convinced of the value of the alternative lifestyle that he encountered with the region's "breakaways" and in the rural communes. Later, in hindsight, he wrote that he had experienced this period as a "Sunday".

Lifestyle

- House small and dingy, but open to everyone
- Sleeps on a door taken off its hinges
- Vegetarian, lives off local farmers
- Gives away money and buys people land left and right
- Drove an old Citroën 2CV to teach at Montpellier (received license after failing his driving license nine times)

(AG, La Clef des Songes)

my "relatives" ... are neither family members nor people of education even of vast culture ... but the poor among the poor"

Professor at Montpellier

"I will never forget the evenings spent at Villecun, the two of us doing math by the light of the old oil lamp. [...] Grothendieck was completely different, other-worldly. Instead of translating things into another language, he thought and spoke directly in the language of modern structural mathematics, which he had contributed greatly to creating." - Yves Ladegaillerie

Teaching philosophy

1. 1973 – 1977: Taught undergrad courses on analytic functions, topology and the fundamental group, and group theory
2. 1977 – 1978: Taught a course on the geometry and combinatorics of polyhedra
3. $\sim$ 1977 – 1982: Taught courses on research

(AG, letter to friend)

The mathematical thought of a child (in so far as it actually leads to a "discovery") could be more "valuable" than a published work (inasmuch as it is mindless and joyless, a routine publication). Or rather, the one is valuable, and the other is spiritual and psychological "junk". ... Polyhedra (take just the cube or even the icosahedron) are an equally inexhaustible source of mathematical reflection and insight on every "level".

Caleb Ji

Part 1: Upbringing (1928–1949)

Part 2: Mathematics (1949–1970)

Part 3: Freedom (1970–1991)

Part 4: Hermitage (1991–2014)

- Tries to stimulate his students toward creativity, with varied success
- Gives a detailed account of this in *La Clef des Songes*

(AG, En Guise de Programme)

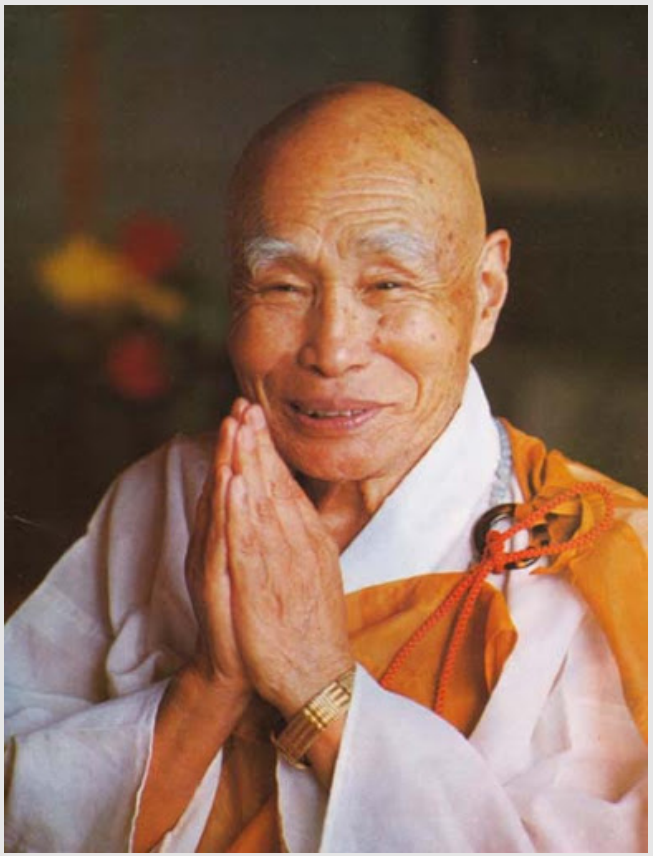
When an intense curiosity animates research, we advance as if carried on impatient wings. Are we not then like a venturesome bark, which avidly plows the immeasurable ocean? Yes, we are surrounded on all sides by billowing mists, ceaselessly taking shape, illuminated by our searching gaze, ceaselessly dissolving in order to challenge us all the more to penetrate them!
[...]
It lies with us whether we will be the child absorbed in a fascinating game – or hopping marionettes...

Encounter with Buddhism: 1974

"Since that memorable day in May when, beneath the midday sun, I perceived a bizarrely dressed person singing in the road while accompanying himself on a drum, and aiming (there could be no mistake about it) directly for my garden where I was busy with some solitary work - since that day, I have had the privilege and the pleasure of seeing many sympathizers and adepts of Guruji pass through my home."

Nichidatsu Fujii

- Founded the activist Buddhist order Nipponzan Myohoji in 1917
- Travelled throughout Japan promoting peace during WWII
- Movement travels the world building peace pagodas, chanting, and calling for peace and non-violence



Encounter with Buddhism

- AG seriously considered to be made Fujii's successor, though not a Buddhist!
- Continued to engage with Buddhists and their practices even into his hermitage

> *"In remembrance of those friends and of their respected Teacher, I still sing the Prayer, whether alone or in a friend's house, before every meal. This I feel is something of great value that remains from my contacts with Fujii Guruji and his followers."*

Na Mu Myo Ho Ren Ge Kyo

Turning inward: 1979

- Spent a year in total solitude at La Gardette, lived with no electricity or running water
- Wrote *Éloge de l'Inceste*
- Delved into parents' correspondence

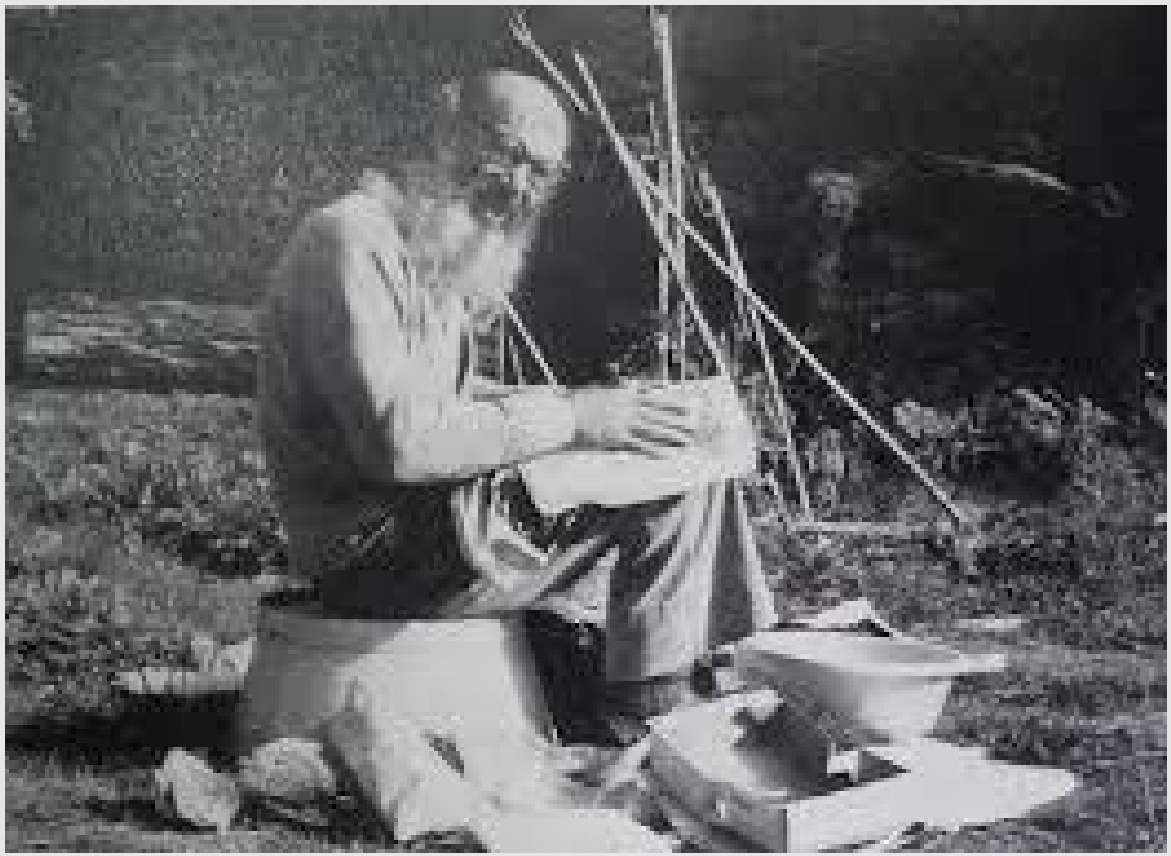


Figure: AG at La Gardette

Parents' correspondence

- Determined the passionate love between his parents to be a myth
- Found that he had been simply abandoned as a baby

Parents' correspondence

(AG, La Clef des Songes)

[...] it is (in 1933) the destruction of the family by the abandonment of the children - wanted by it and imposed, under the banner of the great passion who sanctifies everything, to a subjugated father who ends up saying amen to everything. At the end of this year, when my mother is about to join my father, who has been burning for six months waiting for him in Paris, she appears as the radiant Triumphant, coming to reign supreme over the swooning male - over the hero of yesteryear, fallen, pampered, despised ... This insane apotheosis in the life of my mother, who deeply marked my life, that of my sister as well as that of my mother herself and that of my father, surely marks the most low that they have touched both spiritually [...]

Les Aumettes: 1980-1991

(AG, letter to friend)

"For my part I live a very retired life in a pretty little house, very secluded and rural, surrounded by hills planted with vineyards, at the foot of the Mont Ventoux - one of the most remarkable mountains in Europe. I am thoroughly pleased with my solitude [...] the life of an old eccentric. But an eccentric who is happy to live [...]"

Les Aumettes: 1980-1991

- House found for him by Y., an adventurous woman with whom he had a passionate relationship
- Increasingly in solitude, though continues extensive correspondence
- Alternately writes mathematical and philosophical meditations

(AG, letter to friend)

"In regard to solitude, for me it is more than a choice, it is rather an absolute necessity, and is not imposed on me by some psychological state, rational or otherwise, in any way."

Mathematical meditations

- La Longue Marche à travers la Théorie de Galois (1981, ∼1600 pages)
- Pursuing Stacks (1983, ∼600 pages)
- Letter to Faltings (1983)
- Esquisse d'un Programme (1984)
- Vers une géométrie des formes (1986, ∼600 pages)
- Les Dérivateurs (1990-1991, ∼2000 pages)

Esquisse d'un Programme

Having tired of teaching, Grothendieck applied to the CNRS (basically succesfully) for a research position without teaching, outlining ideas in the following subjects:

- Galois-Teichmüller Theory and Deligne-Mumford stacks
- dessin d'enfants and $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$
- Regular polyhedra, infinite and over finite fields
- New foundations for topology

(AG, Esquisse d'un Programme)

"But today I am no longer, as I used to be, the voluntary prisoner of interminable tasks, which so often prevented me from springing into the unknown, mathematical or not. The time of tasks is over for me. If age has brought me something, it is lightness."

Belyi's theorem and dessins d'enfants

Theorem (Belyi)

A complex algebraic curve can be defined over $\overline{\mathbb{Q}}$ if and only if it can be written as a covering over $\mathbb{P}^1$ ramified over at most three points.

- A dessin d'enfant is simply a graph with vertices colored alternately black and white, on a surface.
- A dessin gives a subgroup of F_2, and thus am algebraic curves over $\mathbb{P}^1$ ramified over at most three points.
- By Belyi's theorem, these are identified with algebraic curves defined over $\overline{\mathbb{Q}}$. This implies there is a $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on these dessins!
- This action can be identified with the mysterious faithful outer action of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on $\widehat{F_2}$, by Grothendieck's theory of étale π_1.
- Bottom line: $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ itself is embedded in how dessins transform!

Anabelian geometry

- Motivated by dessins d'enfants, AG outlined anabelian geometry with a letter to Faltings
- Anabelian question: How much information about the isomorphism class of the variety X is contained in the knowledge of the étale fundamental group?
- Conjecture (proven by Mochizuki): $\pi_1^{\text{et}}(C)$ determines C where C is an appropriate hyperbolic curve.

Figure: Gerd Faltings

Figure: Shinichi Mochizuki

Récoltes et Semailles

- Written 1983-1986, a personal account of AG's life as a mathematician
- Harshly critiques many other mathematicians, particularly Deligne, for a loss of morals
- Claims his work was buried and his vision torn apart

(AG, R&S)

"Yet it is not these gifts, nor the most determined ambition combined with irresistible will-power, that enables one to surmount the "invisible yet formidable boundaries" that encircle our universe. Only innocence can surmount them, which mere knowledge doesn't even take into account, in those moments when we find ourselves able to listen to things, totally and intensely absorbed in child play."

Transition out of math

- At the end of R&S, indicates his plan to return to the topoi, arithmetic geometry, and motives of his SGA days
- Cut short by a deep conviction that such an investment into mathematics was not right for him anymore

(AG, La Clef des Songes)

I knew very well that "I" had the power to end this mathematical frenzy, and even that it was urgent to end it and to meditate. And for as I rejected this increasingly urgent need, I decided to ignore it, to continue to let go of the reins of the pleasure and the vertigo of this "mathematical high". I understood that the mechanism to which I left control over my person, on my soul, was no different than that of the drunk, or the morphine addict.

La Clef des Songes

- A spiritual reflection written in 1987
- Themes: Creativity, repression, dreams, solitude, the Group, creation, love, freedom, self-awareness, self-deception, the nature of God, spirituality, stagnation, religion
- Includes: historical, political, and religious commentary, autobiography and explanation of his life, *Les Mutants*

(AG, La Clef des Songes)

"But above all, the creative path is a solitary path. It is what frightens. And this great fear of creating, this great fear of being yourself, is none other than the fear of being alone, in a world where the only one who is accepted is the one who merges into the herd or who represents it."

Les Mutants

1. C. F. S. Hahnemann (1755-1843): German doctor and savant, modernized the medicine of his time.
2. C. Darwin (1809-1882): English natural scientist; savant.
3. W. Whitman (1819-1892): American journalist, writer and author; poet and savant.
4. B. Riemann (1826-1866: German mathematician; savant.
5. Râmakrishna (1836-1886): Indian (Hindu) preacher, instructor.
6. R. M. Bucke (1837-1902): American doctor and psychiatrist; instructor and annonciateur [herald].
7. P. A. Kropotkin (1842-1921): Russian geographer and savant; anarchist revolutionary.
8. E. Carpenter (1844-1929): English pastor, farmer, thinker and writer, instructor.

Les Mutants (cont.)

9. S. Freud (1856-1939): Austrian doctor and psychiatrist; savant and founder of psychoanalysis, the key to a new scientific humanism.
10. R. Steiner (1861-1925): German savant, philosopher, writer, speaker, pedagogue; visionary instructor, founder of Anthroposophy.
11. M. K. Gandhi (1869-1948): Indian lawyer and politician, instructor, advocated for the spread of ahimsa (non-violence).
12. P. Teilhard de Chardin (1881-1955): French (Jesuit) priest and paleontologist; (Christian) religious ecumenical thinker, mystical visionary, worked for a reconciliation between religion and science.
13. A. S. Neill (1883-1973): English instructor and teacher and pedagogue, who campaigned for an education with freedom.

Les Mutants (cont.)

14. N. Fujii (known as Fujii Guruji) (1885-1985): Japanese buddhist monk; instructor.
15. J. Krishnamurti (1895-1986): Indian religious thinker, speaker and writer; instructor.
16. M. Légaut (1900-1990): University lecturer, farmer, French Christian religious thinker and writer, follower of Jesus of Nazareth, worked for the spiritual renewal of Christianity.
17. F. Carrasquer (1905-1993): Spanish primary schoolteacher and educator; pedagogue and militant anarchist, believed in a "self- determined" school and society.
18. E. Slovik (1920-1945): American worker and employee; apparently without any special calling.

Rudi Bendt

In contrast to the mutants, he remembers Rudi Bendt, a simple man who was even greater simply by his love.

(AG, La Clef des Songes)

Since I happen to meditate on my life and on myself, the action in my life of the love he gave me in my childhood, without even knowing it nor surely want it, underground action, elusive, invisible to all except to God alone, is only beginning to become apparent to me. And when I measure my actions and my failures (and even my successes ...) in terms of who he was, I feel my smallness – not by a commendable effort of modesty, but by the evidence of truth.

Religious and spiritual psychosis

- Begins hearing voices, hallucinating angels and demons
- Identifies with Marthe Robin, a legendary Catholic Franciscan nun
- Believes in the coming of a New Age, which he announces to many of his acquaintances (though he retracts it)
- In 1990, undertakes an epic 45-day fast. Saved by his friends and children

Les Dérivateurs (1990 – 1991)

- Returns to mathematics for a "badly needed" vacation
- Writes $\sim$ 2000 pages of serious math research in under a year
- Returns to the derived categories of the SGA days, to build the foundations better using homotopical algebra
- Starting point is fixing the non-functoriality of the cone construction of derived categories
- Anticipates the infinity categories of today, while also providing its own independent perspective and results

Leaving

- Unexpectedly disappears in 1991 after burning all his papers, except some math he leaves with Jean Malgoire (professor at Montpellier)

Solitude

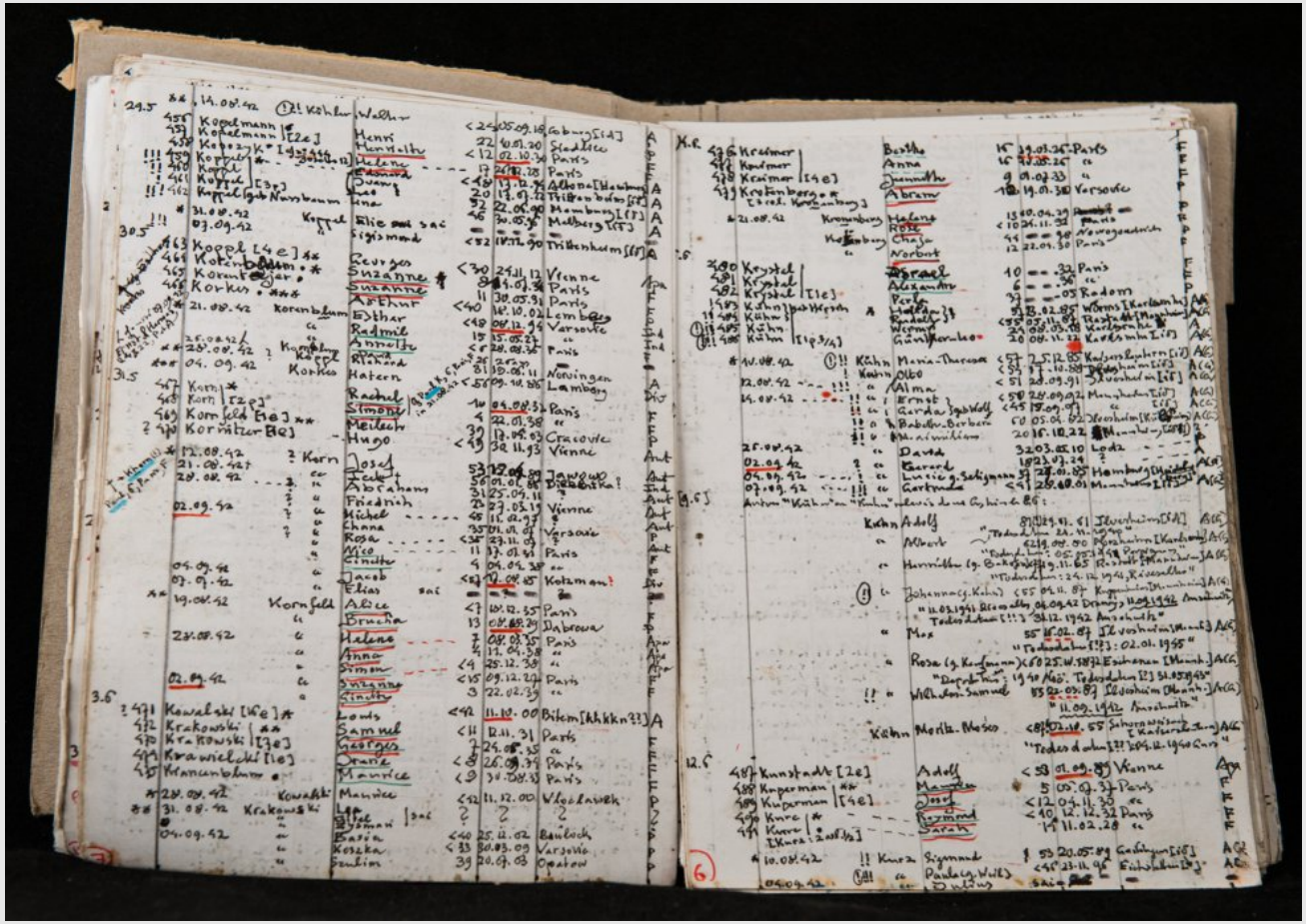
- Extreme isolation in Lasserre

Plants

Cared greatly about plants, growing some using alcohol

The problem of evil

Haunted by the Holocaust, the weight of history of the 20th century, and other manifestations of evil he believed he saw



Visited by grad students

"Hesitantly, I asked him if he would allow me to take a photograph with him. I said I wanted to keep it as a memento. In his gentle manner, he told me to come closer. He shook my hands and hugged me and said that this hug was a better memento." - Mohammad Hadi Hedayatzadeh

Visited by grad students

"[...] when he came back out of his house he presented me with a tomato and a packet of almond paste. The tomato was large and fresh and came from his garden – impressive for January – and he told me to eat it in good health." - Katrina Honigs

Death

- Reunited with his children shortly before his death
- Died in a nearby hospital on November 13, 2014
- Approximately 30,000 pages of writings left behind from Lasserre on math, physics, the psyche, and the problem of evil

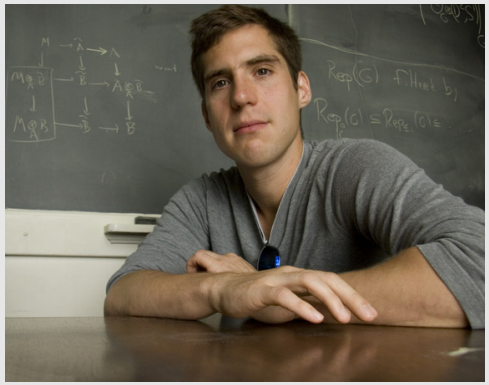
Lasserre writings

- Elementary Schematic Geometry
- The Structure of the Psyche
- Spirit and Creation
- The Problem of Evil
- Reflections on Creation

Acknowledgments

- Biographies by Winfired Scharlau and Leila Schneps, available on grothendieckcircle.org
- Notices of the AMS articles, particularly *Comme Appelé du Néant* by Allyn Jackson
- Centre for Grothendieckian Studies, `https://csg.igrothendieck.org/`, by Mateo Carmona

Some successors



(a) Jacob Lurie

(b) Johan de Jong

(c) Vladimir Voevodsky

(d) Peter Scholze