LIE THEORY

Time and location: Fridays 10:10 am - 12:00 pm in Math 528
Section instructor: Caleb Ji (cj2670@columbia.edu)
Course website: www.math.columbia.edu/~calebji/teaching/lie.html

1. Description of course

The notions of symmetry and geometry are fundamental ideas in mathematics that everyone perceives. However, it was not until the late 19th century with Felix Klein's Erlangen program that their systematic study began, revealing that this subject holds much more than what we intuitively understand from the beginning. Over the course of the next 100 years, mathematicians would pull back the curtain time and time again on the beast of symmetry to reveal more of its intricate, unexpected structure. The study of symmetry in its spatial and geometric form today is generally known as Lie theory, in honor of Sophus Lie. Lie theory now plays a key role in numerous fields of modern mathematics including algebraic geometry, algebraic topology, number theory, and mathematical physics, while also being a major field in its own right.

The primary goal of this course is to explain the basic notions of Lie groups and Lie algebras. A Lie group is what one gets by combining two of the most fundamental concepts in math: groups and manifolds. A manifold that admits a group structure compatible with its geometric structure (e.g. $\mathbb{R}$, tori, elliptic curves, matrix groups) automatically inherits rich structures that are not immediately obvious. Chief among these is its Lie algebra, given by the tangent space at its identity. In this course we will explore the structure of Lie groups and Lie algebras, their highly interesting classification, their representation theory, and their connections to other mathematics. We will also touch on the history of the development of Lie theory.

The material for this class will be drawn from *Lectures on Lie Groups* by Adams [Ada82], [CSM95], [Kir08], [FH91], and other sources. Good historical accounts include [Bor01], [JP21], [Haw00].

2. Prerequisites

There are no official prerequisites to this class. However, it is recommended, but not required, that you have taken at least one 400 level math class at Columbia. In this class we will encounter concepts from abstract algebra, analysis, representation theory, and differential geometry, but we will explain these concepts from the beginning when we introduce them. This class should be thought of as an advanced 400-level course, but the grading is very lenient and there are no problem sets/exams.

3. Grading

This class will meet once a week on Fridays. You are expected to attend and participate in every meeting. Unexcused absences may result in a deduction from your grade. Aside from participation, your grade will either be based on your two presentations. If time constraints do not allow everyone to give two presentations, those who present once will instead write an expository paper on a topic relevant to this class.

Completing either two presentations or a presentation and a paper at a satisfactory level, along with full attendance, will result in an A in the class. Multiple unexcused absences or a presentation/paper that clearly does not live up to the standard of the class will result in an A-. Failing to give a presentation/turn in a paper will result in a grade in the B range. A grade of A+ is generally not given except under extraordinary circumstances.

Presentation. Each presentation should last approximately an hour. There will be two presenters each week, and you are encouraged to prepare with the person you are presenting with and find connections between your topics.

You are highly recommended to use the chalkboard, rather than slides. Your talk should primarily be a math talk, so you should be writing formulas, theorems, and maybe a few proof sketches.

You should write legible notes for your presentation and send them to me to post on the class website. These may be either typed or handwritten and do not need to be formal.

Furthermore, every topic will involve some basic mathematical material that is part of the advanced undergraduate curriculum. You are encouraged to explain these parts in detail. You are also encouraged to sprinkle in relevant historical tidbits if you wish. Finally, you are also free to prepare problems for the class to discuss after you have finished your talk.

Paper. If you only give one presentation, you will turn in an expository paper at any point before the end of the semester. You may choose to delve deeper into the topic you presented on or choose a different topic. The paper needs to be single-spaced, in LaTeX, and at least 4 pages long, not including references. You are again encouraged to weave in historical discussions in your paper as you see fit.

4. Office hours

The primary purpose of office hours is to discuss your presentation/paper with me. I can give you an overview of your topic and answer questions you may have during this time. I will generally have handwritten notes on your topic that you can base your talk on. I will usually be available for thirty minutes before or after class. Please arrange these meetings with me in advance.

References

[Ada82] J. Frank Adams. *Lectures on Lie Groups*. University of Chicago Press, Chicago, 1982.

[Bor01] Armand Borel. *Essays on the History of Lie Groups and Algebraic Groups*, volume 21 of *History of Mathematics*. American Mathematical Society, Providence, RI, 2001.

[CSM95] Roger Carter, Graeme Segal, and Ian Macdonald. *Lectures on Lie Groups and Lie Algebras*, volume 32 of *London Mathematical Society Student Texts*. Cambridge University Press, Cambridge, 1995.

[FH91] William Fulton and Joe Harris. *Representation Theory: A First Course*, volume 129 of *Graduate Texts in Mathematics*. Springer-Verlag, 1991.

[Haw00] Thomas Hawkins. *Emergence of the Theory of Lie Groups: An Essay in the History of Mathematics 1869–1926*. Springer, New York, 2000.

[JP21] Lizhen Ji and Athanase Papadopoulos. *Sophus Lie and Felix Klein: The Erlangen Program and Its Impact in Mathematics and Physics*. Springer, Cham, 2021.

[Kir08] A. A. Kirillov. *Introduction to Lie Groups and Lie Algebras*, volume 113 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2008.